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The Association
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Design Of Multi-Storey Steel Frame Buildings.

By

H. L. HOLLAND, M.Sc.(Eng.), A.M.Inst.C.E.

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SYNOPSIS.

PART I—GENERAL.

Building frames in preference to load-bearing walls for multi-storey buildings. Types of frames—single-storey, multi-storey, single-bay, multi-bay. Preliminary information required by designer—Design Codes and Specifications, loading, column bases and foundations, fire protection. Methods of design—simple design, semi-rigid and fully rigid design. Resistance to wind forces.

PART II—SIMPLE DESIGN.

Simple design of (*a*) a single-bay five-storey steel frame building, and (*b*) a three-bay, nine-storey steel frame building. Roof, floors, roof beams, wall-beams, main floor-beams and connections. Columns, column splices, riveting, column bases. Wind stress analysis of a three-bay, nine-storey building frame. Beams and columns subjected to wind forces. Beam-to-column connections designed to withstand frame moments due to wind.

PART III—FULLY RIGID DESIGN.

Limitations of simple design and alternative design methods. Slope-deflection analysis of a three-bay, nine-storey rigid steel building frame. Welded beam-to-column connections. Economy in steel resulting from rigid design. Suggestions for further study.

DESIGN OF MULTI-STOREY STEEL FRAME BUILDINGS.

By

H. L. HOLLAND, M.Sc.(Eng.), A.M.Inst.C.E.

PART I.

GENERAL.

For buildings of more than four or five storeys, load bearing walls would necessarily become very thick in the lower storeys, so that not only would they be very expensive, but also they would occupy a disproportionate area in plan of the building. For tall buildings, too, it is necessary to investigate the stability under the influence of wind forces, and it is found that for a building of, say, twenty storeys the overturning moment due to wind pressure is very severe, and may be critical. Consequently, when a multi-storey building is projected a frame type of construction is to be preferred, since, in addition to the saving it will afford in space and in materials, it is better able to resist the heavy stresses set up by the vertical loading and by wind forces. Other advantages to be gained from frame construction lie in the reduction possible in construction time, since work may proceed on several wall panels and floors simultaneously, and in the use of lightweight prefabricated wall panels and floors which are particularly suited to multi-storey frame buildings.

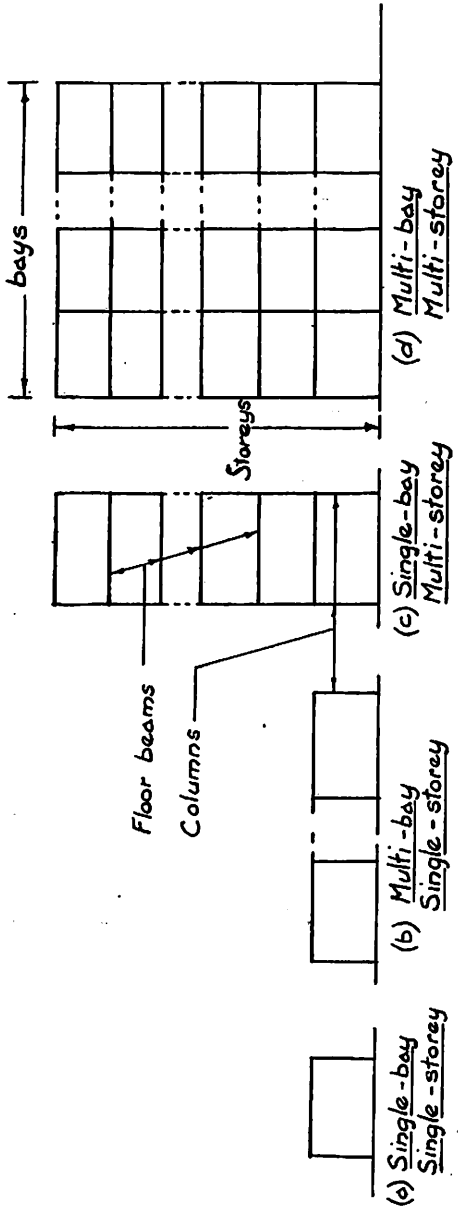
Building frames may be of one or more storeys and of one or more bays, as indicated in Fig. 1. The frame consists of the column members or "stanchions" supporting the beams or girders which, in turn, carry the walls and floors of the building.

It is intended to deal in this pamphlet with the structural design of multi-storey steel building frames employed in the construction of office blocks, warehouses, shops, apartment buildings, hospitals, colleges and the like. The building frames examined in Part II are of one bay and three bays although, in general, the design methods are applicable no matter what the number of bays and storeys.

Preliminary Information Required by Designer.

Before the design for a steel building frame can be prepared certain information must be provided for the designer, viz. :—

(1) Plans indicating the layout of each floor of the building with dimensions of rooms, positions of dividing walls, windows,



(a) Single-bay Single-storey

(b) Multi-bay Single-storey

(c) Single-bay Multi-storey

(d) Multi-bay Multi-storey

Fig 1. Building Frames.

doors, partitions, corridors, lift shafts and stairways, and any special openings.

(2) Elevations indicating the number and heights of storeys, set backs and basements.

(3) Loading. The superimposed loads appropriate to each floor should be specified, with details of any special loads, moving loads, and dynamic or shock loads. Wind forces must be ascertained for the locality, and the possibility of earthquakes must be considered.

(4) Foundation conditions and safe bearing pressures must be determined.

(5) Position of proposed building with regard to building lines, and need for foundations to conform to such building lines.

(6) Details of floors and roof construction, together with information regarding floor and ceiling finishes.

(7) Information concerning wall construction, with details of any special cladding.

(8) Fire-protection requirements.

In particular cases further information to that supplied under the above headings may also be necessary.

Codes and Specifications.

The design of steel building frames, and their behaviour under load have for long occupied the attention of engineers throughout the world. As a result, there is in existence a considerable record of tests on actual frames, and of research work that has been conducted with a view to establishing a rational distribution of steel between the column and beam members of a frame.

In Great Britain codes and specifications have been issued, from time to time, which govern the design of building frames, notably the Building By-Laws of the L.C.C., and the British Standard Specification No. 449, "The Use of Structural Steel in Buildings." It is the intention in this pamphlet to adhere, in the main, to the provisions of the 1948 edition of the British Standard Specification in the examples of design.

Other bodies such as the Building Research Station and the Institution of Structural Engineers* have issued reports on investigations relating to building frames, and the bibliography contains a selection of publications on the subject.

Loading.

The superimposed loads on each floor of a building will depend on the purpose for which the building is intended. For design purposes B.S. 449 enumerates various classes of floor loads, and also ascribes values for horizontal design forces on parapets and balustrades.

In addition to superimposed loads, the dead loads comprising the weights of roofing, flooring, walls, partitions, secondary beams, main beams and girders and stanchions of building frames must be estimated and allowed for in design. It will be seen in the examples of Part II that, by adopting a systematic method of designing each component in turn, the estimating of these weights is reduced to a minimum.

The effects of wind forces on tall buildings and their foundations are very important and must be investigated separately.

Column Bases and Foundations.

The base of a column of a multi-storey building frame may be regarded as hinged, but is usually conceived as fixed in position and capable of resisting bending. Consequently, the foundations, in addition to ensuring that the direct loads from the columns are transferred to the ground without relative settlement, are frequently called upon to restrain the base of the column as it tends to rotate under the action of induced moments.

Fire Protection.

Under the L.C.C. Building By-Laws the columns and beams of building frames must be protected from fire by encasing them in a fire-resisting material. The casing, which may be solid or hollow, is commonly of brickwork or dense concrete, although in recent years there have been reports of the use of asbestos compounds painted or sprayed on to fabric surrounding the steel member, with the object of providing a fire protection that does not add appreciably to the dead load.

From Table 7 of the British Standard Code of Practice CP. 113 (1948) it is seen that a minimum thickness of 2" of solid concrete or brickwork is required to ensure effective fire resistance for two hours.

Methods of Design.

Although the provisions of B.S. 449 are intended primarily with the so-called "Simple Design" procedure in mind, the designer is not restricted to any one method of building-frame design, but is, in fact, encouraged to consider employing alternative methods if by doing so economies will result in the use of steel.

B.S. 449 in Clause 29 refers briefly to three methods of design for steel frameworks, viz. :—

(*a*) Simple design.
(*b*) Semi-rigid design.
(*c*) Fully rigid design.

and it is interesting to note that these three methods of design are referred to in American Specifications.(1)

(*a*) **Simple Design.** In the conventional method of design the main floor beams are assumed to be simply supported by the columns, so that for vertical loading no restraining moments are transmitted through the beam-to-column connections. The columns are designed to support the beam reactions as direct loads and are considered to be subject to bending under the vertical loading, only to the extent imposed by the eccentricities of the beam reactions about both rectangular axes of the column.

(*b*) **Semi-Rigid Design** acknowledges the relief of bending moment on the beams due to the partial restraint of the beam-to-column connections. The connections are designed to exert a known restraining moment on the beams, and in this method of design the effect of the restraining moment on the columns must be investigated.

B.S. 449 directs that where this method is adopted the design shall be in conformity with the "Recommendations for Design" given in the Final Report of the Steel Structures Research Committee, 1936, but stipulates that the connections shall be riveted. It is important to appreciate that the conditions of method (*a*) for simple design rarely exist in practice, since even the simplest web-connections impart a restraint to the beam and cause bending in the columns—an effect to which concrete casing to the beams and columns will contribute. Again, in tall buildings, such restraint at the connections is usually required to ensure that the frame is braced against horizontal wind forces. Thus it is possible with simple design either that the connections will be insufficient to resist the frame moments that are set up, or that the bending imparted to the column members through such connections will prove to be critical. The superiority of method (*b*) then lies in the selection of a beam-to-column connection of known resistance moment, together with the investigation of the effect on the column of the moment transmitted by the connection. In the Final Report are given details of standard connections, together with tabulated values of the restraining moments that they exert at the ends of the beams.

(*c*) **Fully Rigid Design.** In the design of a reinforced concrete building frame it is customary to investigate and to provide for the moments that occur at the joints due to the rigidity of the frame. The same procedure is appropriate in the case of steel building frames where the joints are fully rigid as may be the case with welded connections. Rigid design for multi-storey steel building frames is said to afford appreciable economy in steel.(2)

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(1) The numbers in parenthesis refer to the bibliography at the end of the pamphlet.

Fully rigid design methods have been comparatively little used as yet in the commercial design of multi-storey steel building frames; the chief deterrent, perhaps, being that they require considerably more time in their application than does the simple method. It has been found that attempts to simplify rigid design methods lead to losses in accuracy and consequently in economy to a point where their use can no longer be justified on the grounds of steel saving.

The numerous methods of rigid frame analysis that have been advanced and their development are in themselves an extensive subject quite outside the scope of this pamphlet. In Part III, following the design of a nine-storey, three-bay building frame according to the simple method of design, the same frame is redesigned as a rigid frame by the application of a slope-deflection analysis that was first suggested in 1931. While it is not intended to imply that this method is necessarily superior to other methods, it is advantageous in that it is comparatively rapid in application and is, to a certain extent, self-checking.

Resistance to Wind Forces.

An investigation of the effects of wind forces on a tall building is required to ensure that :—

(*a*) the building is stable as a whole and will not overturn. B.S. 449 stipulates that the building frame must be capable of withstanding the action of the wind forces, but regards general stability computations for wind pressures as unnecessary in the case of buildings for which the height is less than three times the effective width, when stiffening is provided by the walls and floors.

(*b*) the non-vertical wind forces on the building can be transmitted to the foundations without undue swaying of the building and risk of collapse. Wind pressures on a building give rise to forces that must be transferred to the foundations through the floors and framework of the building. Normally, diagonal bracing between the columns of a building frame is not feasible because of the interference it would cause to windows and doorways, and in tall buildings the framework is braced against the horizontal wind loads by ensuring a degree of rigidity or restraint at the beam-to-column connections. This, in turn, will induce bending in both beams and columns which must be investigated.

For design purposes the wind is considered to exert a horizontal pressure on the vertical surface of the building, the intensity of which is related to the wind velocity. According to B.S. 449 (1948), Clause 11, the wind pressure may be regarded as uniform over the exposed vertical surface of the building, and the British Standard recommends values of the wind velocity for use in different parts of the British Isles, together with the equivalent pressures. Tests have indicated that there is in addition a suction effect on a flat roof due to a wind blowing horizontally, which may be regarded as a negative pressure (*i.e.*, vertically upwards) when the general stability of the building is considered.

The usual procedure is to design the frame members first to resist the vertical dead and superimposed loads, and then to investigate the variation in the stresses in these members due to wind forces. In accordance with Clause 25 of B.S. 449 the section adopted for a member from a consideration of vertical dead and superimposed loading is considered to be adequate if the increase in stress due to wind forces is not more than 25 per cent. of the permissible stress for the member. The influence of wind forces on a building frame and its connections will be investigated in Part II.

PART II.

SIMPLE DESIGN.

(A) Design of a Single-Bay, Five-Storey Steel Frame Factory Building.

The building of Design Example (A) is to form a wing of a factory, and for purposes of design the Class VII superimposed floor load from Table 1 of B.S. 449 is adopted. The frames are spaced 15′ 0″ apart and are connected by wall beams and by two floor beams at each floor level, so that the floors are divided into panels as shown in Fig. 7 (Design Sheet A/2).

Roof.—A flat roof is required, and since this will not normally be accessible except for maintenance the design load on the roof is taken as 30 lb./sq. ft. There is little to be gained in varying the roof-slab construction from that employed for the floors, so that the same filler-joist slab will be adopted.

Floors.—The floor construction, as with the roof, may take one of several forms commonly employed such as a reinforced concrete slab or a filler-joist slab. Alternatively, consideration might be given to the use of a patent light-weight fire-resisting floor of hollow bricks or tiles. A detailed consideration of various forms of floor construction would be out of place here, and in Example (A) a 6″ filler-joist slab is employed in accordance with Clause 26 of B.S. 449. The top flanges of the joists will be cut away at each end so that they may be connected to the webs of the wall-beams and subsidiary floor-beams with their top flanges flush. (Fig. 10, Design Sheet A/3).

Floor Beams.—The subsidiary floor-beams Mk. S^1 S^1 frame into the main floor-beams (the roof construction being similar). It is evident from Fig. [illegible], that the main beams of the frame will support a part of the floor slab and superimposed loading directly along their length. However, in design this effect may be ignored and in Design Sheet A/3 the load on the subsidiary floor-beams is assumed to be uniformly distributed. In accordance with simple design procedure the beam moments and reactions are computed as if the beams are simply supported, such end restraint as may be imposed by the connections being ignored. As will be seen later, the beam moments and shears in tall buildings are appreciably modified by wind forces. All floors and floor-beams and connections will be the same throughout the five-storey building.

End Connections.—The end connections for the subsidiary floor-beams Mk. S^1 S^1 consist of cleats in pairs attached to the webs of the beams.

Beam-to-column connections may also consist of web-cleats alone, although it is customary to attach a small shelf-angle to the column to facilitate erection. This shelf-angle may be ignored in the design of the connection, all of the end reaction being transferred through the web-cleats. Alternatively, the shelf-angle may be strengthened to carry the full end reaction itself or in combination with web-cleats (Figs. 2 (*a*) and (*b*)).

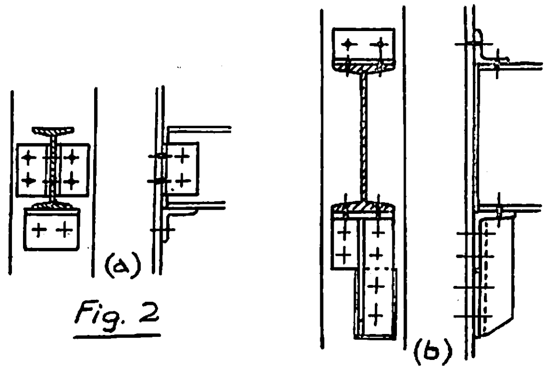


Fig. 2

When a seat connection is designed to carry the full end reaction, the shelf-angle may be stiffened if need be to increase its capacity (Fig. 2 (*b*)). The stiffeners which are machined to fit tightly under the horizontal leg of the angle are proportioned to carry the entire reaction in bearing. Rivets connecting the shelf-angle and stiffener to the column flange in Fig. 2 (*b*) are in single-shear, and since some of them pass through a packing piece their number must be increased in accordance with Clause 41 (*e*) of B.S. 449. A clip-angle is usually provided at the upper flange of the beam in the absence of web-cleats, to hold it in position against any tendency to lateral movement.

The end connections of the main beams in Example (A) (Design Sheets A/2 and A/4) are designed for the vertical end reaction only on the assumption that there is no end moment, although, in fact, there will be a restraining moment developed as the beam deflects. Since the connections are relatively flexible it is to be expected that they will yield, giving rise to a stress distribution that would be impossible to analyse. In tall buildings the beam-to-column connections must also be proportioned to withstand the frame moments due to wind forces in cases where other means of bracing against the action of the wind are not provided. (See Design Sheets B/19 to B/22).

It is good practice in the design of connections to select the cleats so that the bearing value of a rivet on the leg of the cleat is not less than the single-shear value of the rivets employed. The handbooks (3)(4) issued by the firms which produce structural steel provide details of standard web-cleat connections for all sizes of rolled steel joists, that are adequate for the maximum distributed loads on normal spans at specified stresses.

Columns.—In the simple method of design the columns of each storey are treated as "medium columns", that is to say that under an increasing axial load failure of the column would result from buckling in conjunction with the direct stress. The buckling effect is provided for in design by relating the permissible axial compressive stress on the column to its slenderness-ratio given by its effective length divided by its least radius of gyration. In Table 7 of B.S. 449 are given values of the permissible axial compressive stress F_a on a column, obtained from the empirical straight-line formula

$$F_a \text{ (tons/sq. in.)} = 9{\cdot}0 - 0{\cdot}049\, l/r \quad . \; . \; . \; . \; . \quad \text{(i)}$$

for values of the slenderness-ratio between $l/r = 0$, and $l/r = 80$. When l/r exceeds 80 the tabulated values of F_a in Table 7 are obtained from the Perry-Robertson strut formula given in Appendix C of the British Standard, although in practice the slenderness-ratio of a column in any storey of a multi-storey steel frame will rarely exceed 80.

A/1

Design of a Single-Bay Five-Storey Steel-Frame Factory Building

Superimposed Loads

Roof	30 lb/ft²
Floors	100 lb/ft²

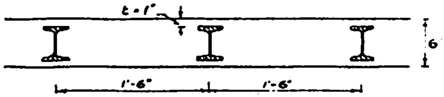


Fig. 3

Roof Slab - adopt Filler-joist slab as for all floors

Floor Slab - say 6" thick with RSJ's at 1'-6" centres. Span 9'-0"

Super load	100 × 1·5'	= 150 lb/ft
Partitions, say	20 × 1·5'	= 30 "
Floor and ceiling finishes, say	20 × 1·5'	= 30 "
Concrete slab	$\frac{6"}{12}$ × 140 × 1·5'	= 105 "
RSJ's self-wt. say		10 "
Total U.D. Load on one joist		= 325 lb/ft

Assume RSJ's to be simply supported, then

Max B.M. = $\frac{325 \times 9.0^2}{2240 \times 8}$ = 1·47 ton-ft.

Max. permissible bending stress (B.S. 449, Clause 26e.)

= (10 + t) = 11·0 tons/in²

Z reqd. = $\frac{1.47 \times 12}{11}$ = 1·60 in³

Adopt 4" × 3" × 10 lb. RSJ's, Z = 3·89 in³

Total load per joist

= $\frac{325 \times 9.0}{2240}$ = 1·31 tons

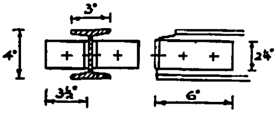


Fig. 4

End reaction = 0·66 tons

Connect to floor beams by means of 2/6" × 3½" × ⅜" cleats, using ⅝" rivets.

Bearing value of 2 No ⅝" rivets on web of RSJ.

= 2/0·688 × 0·24 × 12·0 = 3·96 tons.

Single-shear value of 2 No ⅝" rivets

= 2/$\frac{\pi}{4}$ × 0·688² × 5·0 = 3·71 tons.

Subsidiary Roof-Beams Mk. SS. span 15'-0" at 9'-0" centres.

Super. load (B.S. 449, Clause 10.)	30# × 9·0' =	270 lb/ft.
Slab (including RSJ.s)	$\frac{6"}{12}$ × 150# × 9·0' =	675 "
Bituminous finish	10# × 9·0' =	90 "
Ceiling finish	5# × 9·0' =	45 "
Beam self-wt. say		30 "
Casing	$\frac{9"}{12}$ × $\frac{5"}{12}$ × 140# =	44 "
Total U D Load on beam		= 1,154 lb/ft

Max B M = $\frac{1154 \times 15.0^2}{2240 \times 8}$ = 14·50 ton-ft

At F_b = 10 tons/in², Z req'd = $\frac{14.50 \times 12}{10}$ = 17·40 in³

Adopt an 8" × 5" × 28 lb. RSJ A = 8·28 in² Z = 22·42 in³

A/2

Subsidiary Roof-Beams Mk SS (Cont'd)

Total load on beam

$= \frac{1154 \times 15}{2240}$ = 7 74 tons

End reaction = 3 87 "

Connect to main roof beams by means of 2/6"×3½"×⅜" cleats using ¾" rivets (Fig 5)

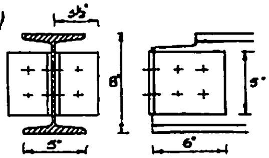


Fig. 5

Edge Roof-Beams Mk. W W.

Span 15'-0"

Super. load	30#×5·0' =	150 lb./ft.
Slab.	$\frac{6"}{12}$ × 150#×4·12' =	309 "
Bituminous finish	10#×4·12' =	41 "
Ceiling finish	6#×4·12' =	21 "
Parapet.	$\frac{10"}{12}$ × 3·0'×150# =	376 "
Beam, self-wt. say.		30 "
Casing.	=	44 "
Total U.D. Load on beam	=	970 lb./ft.

Adopt an 8"×5"×28 lb. RSJ. as for subsidiary roof beams.

Total load on beam

$= \frac{970 \times 15}{2240}$ = 6·50 tons

End reaction = 3·25 "

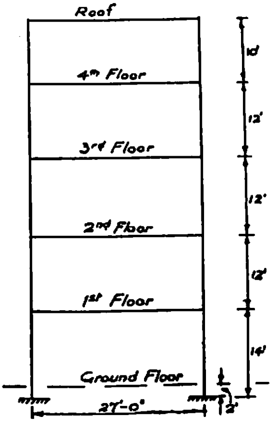


Fig. 6.

Main Roof-Beams. Mk. BB.

Span 27'-0.

Reaction from subsidiary roof beams = 7·74 tons.

Beam, self-wt. say		60 lb./ft.
Casing.	$\frac{10}{12} \times \frac{15"}{12}$ × 140# =	146 "
Total U.D. Load on beam	=	206 lb./ft.

Max. B.M. $= 7{\cdot}74 \times 9{\cdot}0' + \frac{206 \times 27^2}{2240 \times 8}$ = 78·05 ton-ft.

At f_b = 10 tons/in²

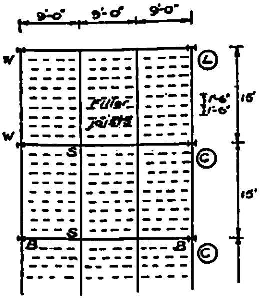
Z req'd. $= \frac{78{\cdot}05 \times 12}{10}$ = 93·66 in³.

Adopt an 18"×6"×55 lb RSJ.

A = 16·18 in² Z = 93·53 in³.

Total load on beam

$= 2 \times 7{\cdot}74 + \frac{206 \times 27}{2240}$ = 17 96 tons



Roof and Floor Plan

Fig. 7

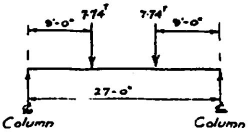


Fig 8

End reaction = 8 98 tons

Connect to columns by means of 2/3½"×3½"×⅜" L cleats using ¾" rivets, (Fig 9) with a 3½"×3½"×⅜ shelf-angle

HI

A/3

Main Roof-Beam Mk BB (Cont'd.)

Bearing value of 4 Nº ¾" rivets

$= 4/0{\cdot}813 \times 0{\cdot}42 \times 12{\cdot}0 = 16{\cdot}35$ tons

Single-shear value of 8 Nº ¾" rivets

$= 8/\frac{\pi}{4} \times 0{\cdot}813^2 \times 5{\cdot}0 = 20{\cdot}72$ tons

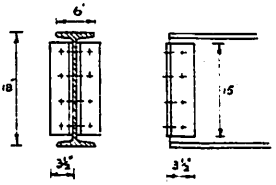


Fig 9.

Subsidiary Floor-Beams Mk. S'S'

Span 15'-0" at 9'-0" centres.

Super load	$100^{\#} \times 9{\cdot}0'$	= 900 lb/ft
Partitions.	$20^{\#} \times 9{\cdot}0'$	= 180 "
Floor and ceiling finishes	$20^{\#} \times 9{\cdot}0'$	= 180 "
Slab (including RSJ.s)	$\frac{6''}{12} \times 150^{\#} \times 9{\cdot}0'$	= 675 "
Beam self-wt. say		30 "
Casing	$\frac{9''}{12} \times \frac{7''}{12} \times 140^{\#}$	= 61 "
Total U.D. Load on beam		= 2026 lb./ft.

Max. B.M. $= \dfrac{2026 \times 15{\cdot}0^2}{2240 \times 8} = 25{\cdot}45$ ton-ft.

At $F_b = 10$ tons/in² – Z req'd. $= \dfrac{25{\cdot}45 \times 12}{10} = 30{\cdot}53$ in³

Adopt a 12"×5"×32 lb. RSJ. A = 9·45 in² Z = 36·84 in³

Total load on beam $= \dfrac{2026 \times 15'}{2240} = 13{\cdot}58$ tons

End reaction = 6·79 "

Connect to main beams by means of 2/3½"×3½"×⅜" cleats using ¾" rivets.

Bearing value of 3 Nº ¾" rivets on web of beam

$= 3/0{\cdot}813 \times 0{\cdot}35 \times 12{\cdot}0 = 10{\cdot}22$ tons

Single-shear value of 4 Nº ¾" rivets

$= 4/\frac{\pi}{4} \times 0{\cdot}813^2 \times 5{\cdot}0 = 10{\cdot}38$ "

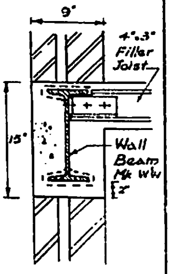


Fig. 10.

Wall-Beams. Mk. W'W'. (All as for 1st Floor.)

Span: 15'-0".

Super. load	$100^{\#} \times 4{\cdot}12'$	= 412 lb/ft.
Partitions, say.	$20^{\#} \times 4{\cdot}12'$	= 82 "
Floor and ceiling finishes	$20^{\#} \times 4{\cdot}12'$	= 82 "
Slab (including RSJ.s)	$\frac{6''}{12} \times 150^{\#} \times 4{\cdot}12$	= 309 "
External wall	$\frac{9''}{12} \times 12{\cdot}0' \times 140^{\#}$	= 1260 "
Beam self-wt. say		30 "
Casing (included in wt. of external wall)		Nil
Total U.D. Load on beam		= 2175 lb./ft.

Max. $= \dfrac{2175 \times 15{\cdot}0^2}{2240 \times 8} = 27{\cdot}30$ ton-ft

At $F_b = 10$ tons/in². Z req'd. $= \dfrac{27{\cdot}30 \times 12}{10} = 32{\cdot}76$ in³

Adopt a 12"×5"×32 lb RSJ A = 9·45 in² Z = 36·84 in³

Total load on beam $= \dfrac{2175 \times 15'}{2240} = 14{\cdot}56$ tons

End reactions = 7·28 "

End connections as for subsidiary floor-beams

HH

A/4.

Main Floor Beams. Mk. B'B'.

Span: 27'-0" at 15'-0" centres

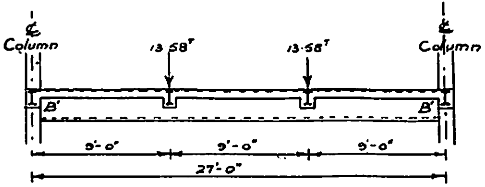


Fig. 11.

Reaction from subsidiary floor beams 13·58 tons.

Beam, self-wt. say 100 lb/ft

Casing $\frac{12"}{12} \times \frac{17"}{12} \times 140^{\#} - \underline{198}$ "

Total U.D. Load on beam - 298 "

Max B.M. - $13{\cdot}68 \times 9{\cdot}0' + \frac{298 \times 27^2}{2240 \times 8} - 134{\cdot}36$ ton-ft.

At F_b - 10 tons/in². Z requ'd. $- \frac{134{\cdot}35 \times 12}{10} - 161{\cdot}2$ in³

Adopt a 20"× 7½" × 89 lb. RSJ. A - 26·19 in² Z - 167·29 in³

Total load on beam $- 2 \times 13{\cdot}58 + \frac{298 \times 27}{2240} -$ 30·74 tons.

End reaction - 15·37 "

Connect to column flanges by means of 2/3½"×3½"×½" cleats, using ¾ rivets, (Fig. 12.) with a 3½"×3½"×⅜" shelf-angle.

Bearing value of 5 No ¾" rivets on web of beam - 5/0·813×0·60×12·0 - 29·2 tons.

Single-shear value of 10 No ¾" rivets

$= 10/\frac{\pi}{4} \times 0{\cdot}813^2 \times 5{\cdot}0$ - 25·9 tons.

(Stresses of 12·0 tons/in² for bearing on shop-driven rivets, and 5·0 tons/in² for shear on field-driven rivets are those permitted under B.S.449. Cl.23.)

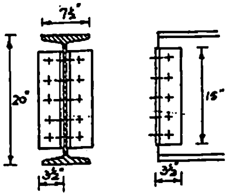


Fig. 12.

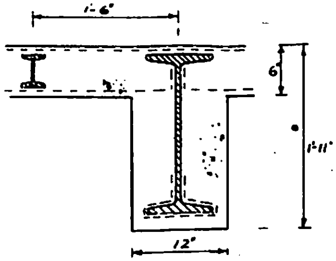


Fig. 13.

The value assigned to the effective length l in a particular case depends on the degree of fixity at the ends of the column, and this is very much a matter of judgment on the part of the designer. The range of end conditions for the columns of building frames are illustrated in Appendix E of B.S. 449, and corresponding values of l recommended for design in terms of the actual length L between points of lateral support are given in Table 15 of the British Standard.

A main column or stanchion of a multi-storey building frame is usually continuous from the foundation to the roof level although not necessarily of uniform section throughout. It is common for the stronger axes of the columns to be aligned at right-angles to the main frames of the building, as is the case in both Design Examples (A) and (B). This practice is, however, by no means universal. Architectural considerations may lead to the columns being placed with their stronger axes in line with the main frames and it is sometimes the case for the external columns of a multi-bay frame to be arranged with their major axes at right-angles to those of internal columns of the same frame (Fig. 14).

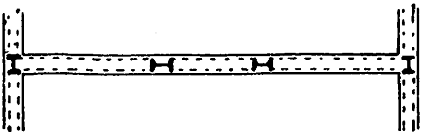


Fig. 14.

The loading (apart from the self-weight of the column and any casing) is transferred to the column from the beams that frame into it at each floor, and such loading is generally eccentric due to the fact that the beam-to-column connections are made at the outer face of the flanges or to the web of the column. The eccentricity of the loading gives rise to bending stresses in the column as well as direct compression, with the result that the capacity of the column is decreased as compared with an axially loaded column. The simple method employed in Design Example (A) ignores bending of the column due to restraint at the beam-to-column connections.

The simple design procedure for proportioning a column is to choose a trial section first of all on the basis of the direct compression, and to investigate the resultant stresses due to the direct compression and bending caused by the eccentric beam reactions on the column. According to Clause 22 of B.S. 449 a column subject to both axial compression and compression due to bending should be proportioned such that

$$\frac{f_a}{F_a} + \frac{f_{bc}}{F_{bc}} \not> 1 \quad \dots \quad \dots \quad \text{(ii)}$$

where f_a = the axial compressive stress

F_a = the permissible compressive stress in the column when subject to axial loading only

f_{bc} = the sum of the compressive stresses due to bending about both rectangular axes of the column section and

F_{bc} = the permissible compressive bending stress on a member

The equation of condition (ii) above may be rewritten in the general form :—

$$\frac{P}{F_a\,.A} + \frac{M.c}{F_{bc}\,.I} \ngtr 1$$

or

$$\frac{P}{F_a.A} + \frac{M.c}{F_{bc}.A.r^2} \ngtr 1$$

whence the required area of the column-section

$$A \nless \frac{P}{F_a} + \frac{M.c}{F_{bc}.r^2} = A_a + A_{bc}$$

indicating that the required area A must be not less than the sum of the areas required for direct load (A_a) and bending (A_{bc}).

The process of selecting a section and checking may need to be repeated several times before the most satisfactory section is determined.

In calculating the loading on the column in Design Sheet A/5, no reduction has been made in the total superimposed loading incident on the column at each descending floor level. B.S. 449, in Clause 8, acknowledges that it is unlikely that the full design load will occur on all floors simultaneously by permitting a gradual reduction in the superimposed loading as each storey length of the column is considered from the roof down. However, Design (A) is for a factory building wherein it is likely that some floors will be used for storage and machinery employed. In Design Example (B) advantage is taken of the permissible reduction of floor loads in column design.

It is evident from Design Sheets A/5-A/8 that whereas the preliminary trial for the column section must commence at the base where the load is heaviest and work upwards to establish a suitable tapering of the column, the final checking is conveniently worked through from roof to base, immediately above each floor. Where the change in column section occurs at alternate floors it is only necessary (when the loading on all floors is the same or very nearly so) to check that the section is suitable at the levels of the splices. Thus, in Example (A) it would suffice if the check on the column

A/5.

Columns Mk. C.

Class VII loading from B.S. 449, Table 1. No reduction made in total superimposed load on columns

Direct Load on Column:

From main roof beams	8·98 tons.
" edge " "	6·50 "
Self-wt. and casing, say	0·50 "
Load above 4th Floor	= 15·98 tons
From main beams (4th Floor)	15·37 "
" wall " "	14·56 "
Self-wt. and casing, say	0·50 "
Load above 3rd Floor	= 46·41 tons
From main beams (3rd Floor)	15·37 "
" wall " "	14·56 "
Self-wt. and casing, say	0·75 "
Load above 2nd Floor	= 77·09 tons
From main beams (2nd Floor)	15·37 "
" wall " "	14·56 "
Self-wt. and casing, say	0·75 "
Load above 1st Floor.	= 107·77 tons
From main beams (1st Floor)	15·37 "
" wall " "	14·56 "
Self-wt. and casing, say	1·00 "
Load above base.	= 138·70 tons

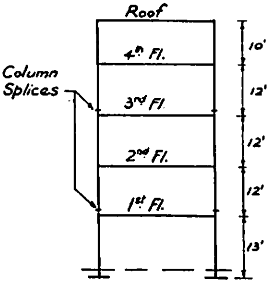


Fig. 15.

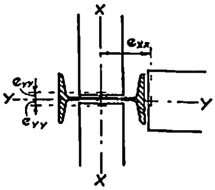


Fig. 16

Preliminary trial with a basic 12"×6" RSJ provides too great a reserve of strength in the upper storeys.

Try a 10"×6"×40 lb. RSJ with 2/9"×¾" plates in the 1st storey tapering to a plain 10"×6"×40 lb. RSJ above the 3rd floor.

Above 4th Floor.

Assume a 10"×6"×40 lb. RSJ to be sufficient in both 5th and 4th storeys.

$A = 11·77 \text{ in}^2$

$I_{xx} = 204·80 \text{ in}^4$

$Z_{xx} = 40·96 \text{ in}^3$

$r_{xx} = 4·17 \text{ in.}$ $r_{yy} = 1·36"$

With concrete casing (B.S. 449. Cl. 18.b.) Min. $r_{yy} = 0·2(b+4) = 2·0"$

For 5th storey columns $L = 120"$, $l = 120 \times 0·85 = 102"$

and $\frac{l}{r} = \frac{102}{2·0} = 51.$ for which $f_a = 6·52 \text{ tons/in}^2$

Direct load = 15·98 tons "

A/6.

Columns Mk.C (Cont'd.)

and $f_a = \dfrac{15.98}{11.77} = 1.36$ tons/in².

Eccentricities of reactions from beams on column at 4th floor:

$e_{xx} = 5'' + \tfrac{1}{2}''$ (say) $= 5.5''$ $\qquad e_{yy} = 0$

Column bending moment due eccentric loading at 4th floor is shared equally between the 5th and 4th storey column-lengths (B.S. 449. Cl. 37b.)

$M_{xx} = \tfrac{1}{2}(15.37 \times 5.5) = 42.2$ ton-ins.

Bending moment on 5th storey columns at roof

$= 8.98 \times 5.5 = 49.4$ ton-ins.

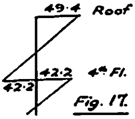


Fig. 17.

So that on 5th storey columns the moment at roof-level governs.

then $f_{bc} = \dfrac{49.4}{40.96} = 1.20$ tons/in².

and $\dfrac{f_a}{F_a} + \dfrac{f_{bc}}{F_{bc}} = \dfrac{1.36}{6.52} + \dfrac{1.20}{10.0} < 1.$

Above 3rd Floor

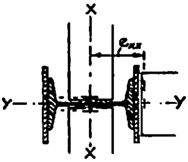
Assume a 10" × 6" × 40 lb. RSJ to be sufficient in the 4th storey and a 10" × 6" × 40 lb. RSJ with 2/9" × 3/8" plates in the 3rd storey.

For 4th storey columns $L = 144''$ $\qquad l = 144 \times 0.70 = 101''$

and $\dfrac{l}{r} = \dfrac{101}{2.0} = 50.5$ for which $F_a = 6.54$ tons/in²

Direct load = 46·41 tons.

and $f_a = \dfrac{46.41}{11.77} = 3.95$ tons/in².



A column splice occurs immediately above the 3rd floor, and eccentricity of load at 3rd floor:-

$e_{xx} = 5'' + \tfrac{3}{8}'' + \tfrac{1}{2}''$ (say) $= 5.88''$

Fig. 18.

For 4th storey columns $\dfrac{I_{xx}}{L} = \dfrac{204.80}{144} = 1.42$

" 3rd " " $\dfrac{I_{xx}}{L} = \dfrac{386.51}{144} = 2.69$

Bending moment in 4th storey due to eccentricity of loads at 3rd floor $M_{xx} = \dfrac{1.42}{1.42 + 2.69}(15.37 \times 5.88) = 31.2$ ton-ins. $\qquad M_{yy} = 0$

$f_{bc} = \dfrac{31.2}{40.96} = 0.76$ tons/in².

and $\dfrac{f_a}{F_a} + \dfrac{f_{bc}}{F_{bc}} = \dfrac{3.96}{6.54} + \dfrac{0.76}{10.0} = 0.681$ - accepted!

Above 2nd Floor.

Assume a 10" × 6" × 40 lb. RSJ with 2/9" × 3/8" plates to be sufficient in both 3rd and 2nd storeys.

$A = 11.77 + 6.75 = 18.52$ in².

W.

Columns Mk. C. (Cont'd.)

$I_{xx} = 204.80 + 181.71 = 386.51 \text{ in}^4$

$Z_{xx} = \frac{386.51 \times 2}{10.76} = 72.0 \text{ in}^3$

Due to concrete casing Min. $r_{yy} = 0.2(b+4) = 2.6''$

For 3rd storey columns $L = 144''$ $l = 144 \times 0.7 = 101''$

and $\frac{l}{r} = \frac{101}{2.6} = 38.9$ for which $F_a = 7.11$ tons/in.²

Direct load = 77.09 tons

and $f_a = \frac{77.09}{18.52} = 4.16$ tons/in.²

Bending moment in 3rd storey columns due to eccentricity of loads at 2nd floor:

$M_{xx} = \frac{1}{2}(15.37 \times 5.88) = 45.2$ ton.-ins. $M_{yy} = 0$

$f_{bc} = \frac{45.2}{72.0} = 0.63$ tons/in.²

and $\frac{f_a}{F_a} + \frac{f_{bc}}{F_{bc}} = \frac{4.16}{7.11} + \frac{0.63}{10.0} = 0.648$ - accepted!.

Above 1st Floor.

Assume a 10"x 6"x 40 lb. RSJ with 2/9"x 3/8" plates to be sufficient in the 2nd storey and a 10"x 6"x 40 lb. RSJ with 2/9"x 3/4" plates in the 1st storey.

For 2nd storey columns $L = 144''$ $l = 144 \times 0.7 = 101''$

and $\frac{l}{r} = 38.9$ as before, so that $F_a = 7.11$ tons/in.²

Direct load = 107.77 tons

and $f_a = \frac{107.77}{18.52} = 5.82$ tons/in.²

A column splice occurs immediately above the 1st floor, and eccentricity at 1st floor:

$e_{xx} = 5'' + \frac{3}{4}'' + \frac{1}{2}''$ (say) $= 6.25''$.

For 2nd storey columns $\frac{I_{xx}}{L} = \frac{386.51}{144} = 2.69$

" 1st " " $\frac{I_{xx}}{L} = \frac{595.49}{180} = 3.31$

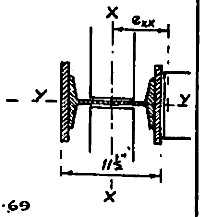


Fig. 19.

Bending moment in 2nd storey columns due to eccentricity of loads at 1st floor:

$M_{xx} = \frac{1}{2}(15.37 \times 6.25) = 48.0$ ton-ins. $M_{yy} = 0$

$f_{bc} = \frac{48.0}{72.0} = 0.67$ tons/in.²

and $\frac{f_a}{F_a} + \frac{f_{bc}}{F_{bc}} = \frac{5.82}{7.11} + \frac{0.67}{10.0} = 0.885$ - accepted!.

Base to 1st Floor

Assume a 10"x 6"x 40 lb RSJ with 2/9"x 3/4" plates to be sufficient

$A = 11.77 + 13.50 = 25.27 \text{ in}^2$

A/8.

Columns Mk. C (Cont'd.)

$$I_{xx} = 204{\cdot}80 + 390{\cdot}69 = 595{\cdot}49 \text{ in}^4$$

$$Z_{xx} = \frac{595{\cdot}49 \times 2}{11{\cdot}50} = 103{\cdot}70 \text{ in}^3$$

For 1st storey columns $L = 180''$ $\quad l = 180 \times 0{\cdot}85 = 153''$

and $\frac{l}{r} = \frac{153}{2{\cdot}6} = 58{\cdot}8$ for which $F_a = 6{\cdot}15$ tons/in²

Direct load = 138·70 tons

and $f_a = \frac{138{\cdot}70}{25{\cdot}27} = 5{\cdot}50$ tons/in².

Bending moment in 1st storey columns due to eccentricity of loads at 1st floor:

$$M_{xx} = \tfrac{1}{2}(15{\cdot}37 \times 6{\cdot}25) = 48{\cdot}0 \text{ ton-ins.}$$

$$f_{bc} = \frac{48{\cdot}0}{103{\cdot}70} = 0{\cdot}46 \text{ tons/in}^2$$

and $\frac{f_a}{F_a} + \frac{f_{bc}}{F_{bc}} = \frac{5{\cdot}50}{6{\cdot}15} + \frac{0{\cdot}46}{10{\cdot}0} = 0{\cdot}941$, – accepted!

Column Splices and Riveting.

Splices to occur above 1st and 3rd Floors. Ends of column-lengths joined by splice to be machined square and faced for bearing over whole area (B.S. 449, Cl. 50 b.)

Adopt ¾" dia. rivets throughout.

Thickness of flange plates above 1st floor ⅜", so that in conformity with Clause 42a. of B.S. 449 a normal rivet pitch of 16 × ⅜" 6" will be adopted. This pitch to be reduced to 3" in splices and in region of beam-to-column connections (Clause 42 b.)

Length of splice plates above and below plane of joint to be not less than 9" – the width of flange (Clause 50 b.)

Adopt 24" × 9" × ⅜" flange splice plates for both splices with 15" × 9" × ⅜" packing pieces. (Fig. 21).

section were made immediately above the 3rd and 1st floors in addition to a check at the base of the column.

The concrete casing around the H-section columns (provided it is of a 1 : 2 : 4 mix or richer, and is of solid section and reinforced as indicated in Fig. 20) will increase their load-carrying capacity (see Eq. (i)) according to the extent that the radius of gyration about the weak axis obtained from $r = 0.2\ (b+4)$, exceeds the minimum value of r for the plain section. A limit to the maximum value of F_a for the cased section is however set in B.S. 449 at 50% above the value for the plain section. A similar relationship due to concrete casing is given for the maximum permissible compressive stress due to bending F_{bc}, and the maximum values of both F_a and F_{bc} should be checked for all cased columns designed in accordance with B.S. 449. Thus, on Design Sheet A/5, the least radius of gyration for the plain 10″ × 6″ × 40 lb. R.S.J. column is $r_{yy} = 1.36''$ for which

$$l/r = 102/1.36 = 75 \text{ and } F_a = 5.36 \text{ tons/in.}^2$$

An increase of 50% would give $F_a = 5.36 + 2.68 = 8.04$ tons/in.^{2}, and since the value of F_a for the cased section of 6.52 tons/in.^{2} is less than F_a + 50% for the plain section it is the correct value to adopt. For the cased section the minimum value of $r_{yy} = 0.2\ (b+4) = 2.0''$, and hence $r_{xx}/r_{yy} = 4.17/2.00 = 2.18$, for which from Clause 19 (*c*) of B.S. 449 (1948) the bending stress factor $K_1 = 1.50$, so that F_{bc} is equal to the lesser of

$$10.0 \text{ tons/in.}^2, \text{ or } \frac{1{,}000}{l/r} \times K_1, \textit{i.e.}, \frac{1{,}000 \times 1.50}{51} = 29.4 \text{ tons/in.}^2$$

The eccentricities of the beam reactions depend on the type of connection (see Table 16, Clause 37 (*a*) of B.S. 449), and in Design Example (A) the eccentricity e_{xx} is assumed to be equal to half the

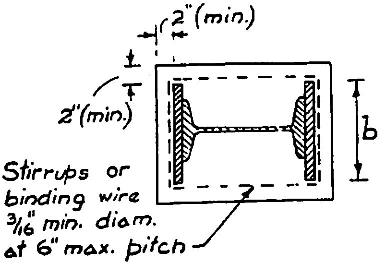


Fig. 20

depth of the column section at each beam connection plus a nominal ½″. The division of the eccentricity moments between the column lengths above and below the beam is made in accordance with Clause 37 (*b*) of the British Standard.

Column Splices.—A convenient length of column for handling during erection will often extend through two storeys, so that splices occur just above alternate floors. For example, the position of the splices in the five-storey building frame of Example (A) are arranged to occur at approximately 15 inches above the top flanges of the beams at the 1st and 3rd floors. The distance between column splices in the lower storeys of a tall building may be restricted to the single storey height, since the weight per foot of the columns will be much greater here than in the upper storeys.

The direct compressive stress at a splice in the column of Example (A) is in excess of the tensile stress due to bending, so that compression will exist over the full section. When this is so, and providing the two abutting ends are machined square to ensure full bearing at the splice, the splice plates are required only to hold the two lengths of column in line, and their thickness need be no greater than that of the thinner of the column flange plates at the splice. The length of the splice plates above and below the plane of the joint, according to Clause 50 (*b*) of B.S. 449, should be not less than the width of the flange at the splice.

Packing pieces are used to make up any difference in the depth of the section of the two column lengths joined at a splice, and such packings may be extended beyond the splice plates on the lighter column length and attached in the shop by means of the first shop rivets (Fig. 21, Design Sheet A/9).

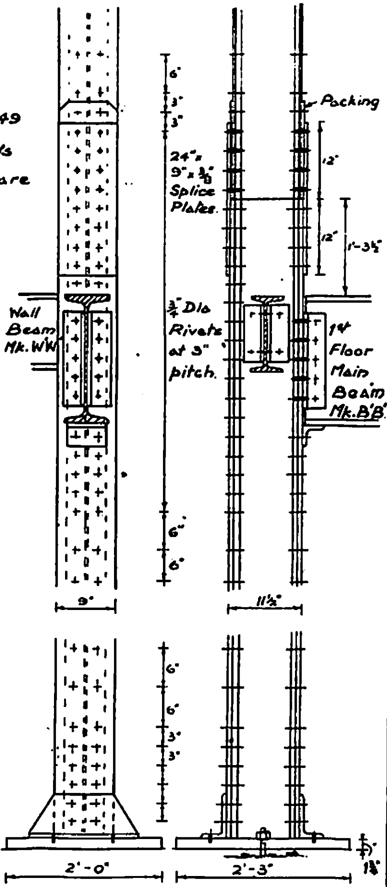
Riveting.—The pitch of the rivets attaching flange plates to the joist of a column is governed by Clause 42 of the British Standard. In Design Example (A) the normal pitch is accordingly 6″ and is reduced to 3″ in the region of the beam-to-column connections and in the splices.

Column Bases.—The dead load on the windward column will provide a stability moment well in excess of the overturning moment due to wind forces, and it is reasonable to assume that the base of the column will distribute the load uniformly, so that a slab base is all that is required. The load is transmitted directly by the bearing of the machined end of the column on the slab, so that the size of the angle-cleats and riveting are not subject to special calculations in this case. In Design Example (B) a gusseted column base will be designed to withstand the end moment in the column due to wind forces as well as the direct load.

A/9

Column Base

The columns are to be provided with a slab base proportioned in accordance with Clause 50c. (ii) of B.S. 449. End of column with angle cleats attached to be machined square so that bearing is true over entire area.


Packing
6"
3"
3"
24" × 9" × 3/8" Splice Plates.
12"
12"
1'-3½"
Wall Beam Mk. WW
3/4" Dia Rivets at 3" pitch.
14" Floor Main Beam Mk. B'B'
6"
6"
9"
11½"
6"
6"
3"
3"
2'-0"
2'-3"
1¾"

Load on base $= 138{\cdot}78$ tons

At pressure of 40 tons/ft.² on concrete foundation:

Area of slab base $\nless \dfrac{138{\cdot}78}{40} = 3{\cdot}47\ \text{ft}^2$

Adopt a 2'-3" × 2'-0" slab, area $= 4{\cdot}50\ \text{ft}^2$

and bearing pressure $w = \dfrac{138{\cdot}78}{4{\cdot}5 \times 144}$

$= 0{\cdot}215$ tons/in²

Slab thickness $t = \sqrt{\dfrac{3w}{F_b}\left(A^2 - \dfrac{B^2}{4}\right)} = 3 \times 0{\cdot}215\left(7{\cdot}76^2 - \dfrac{7{\cdot}50^2}{4}\right)$

$= 1{\cdot}57"$ say $1\tfrac{3}{4}"$

Connect column to base plate by means of 7" × 4" × 5/8" ∠ cleats and 4 No 1" dia stud screws after plate is set in position

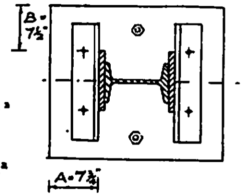
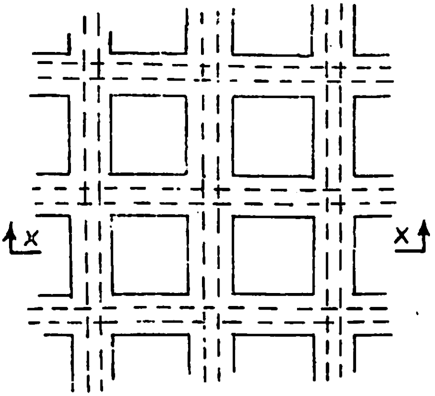


Fig. 21

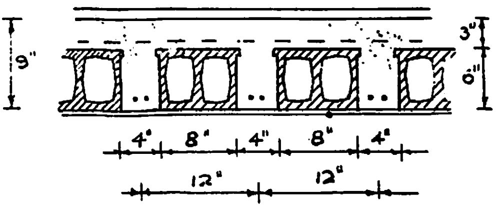
(B) Design of a Three-Bay, Nine-Storey Steel Frame College Building.

The superimposed load adopted for classrooms in Design Example (B) is 100 lb. per sq. ft. and this loading is also applied to the centre bay which provides a hallway at each floor. For the roof design a superimposed load of 60lb. per sq. ft. is employed, the intention being that the roof will be used from time to time for College activities. Two elevators are to be accommodated in the centre of the building as indicated in Fig. 25 which shows a line plan of the roof beams. Fig. 24 is an outline of a main frame of the building, and indicates the widths of the bays and heights of storeys.

Roof and Floors.—The roof and floors are of a reinforced concrete T-beam and hollow tile construction as indicated in Fig. 22. The panels span 20 ft. in each direction in the outer bays.



Plan.



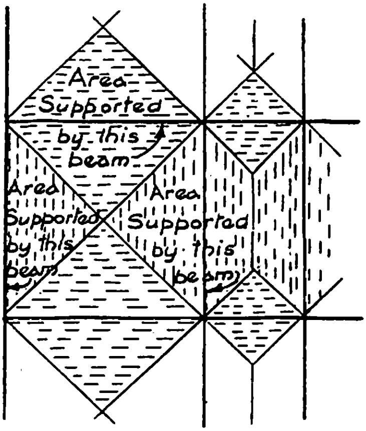
Section XX

Fig. 22.

Beams.—It is assumed that the distributed dead and superimposed loads on the square panels are divided among all the supporting beams in proportion to the tributary areas of the panel formed by the centre-lines of the beams and the 45-degree lines from the corners of the panels as indicated in Fig. 23. In the case of the rectangular centre bay panels the distributed loading on the longitudinal beams Mk. F51, F52, F53, etc., is modified as shown. Thus, on Design Sheet B/5, the panel load for the main beams Mk. F1, F2, F3, etc., is equal to the total uniformly distributed load of 200 lb. per sq. ft. multiplied by the tributary areas of two adjacent panels, viz :—

$$2/200\# \times \frac{10' \times 20'}{2} = 40{,}000 \text{ lb.}$$

The bending moment on the beam is composed of two quantities representing $\frac{W_1.L}{6}$ for the triangular panel loading and $\frac{W_2.L}{8}$ for the load distributed uniformly along the beam consisting of the internal wall and the casing to and self-weight of the beam.


Area Supported by this beam
Area Supported by this beam
Area Supported by this beam

Distribution of Panel Loads

Fig. 23.

Columns.—The design of the columns in Example (B) is carried through in a similar manner to that of Example (A) except that in each storey the superimposed loading on the columns is reduced in accordance with Clause 8 of the British Standard (see Design Sheets B/7, B/12, and B/14) and the column sections are checked at the

B/1

Design of a 3-Bay 9-Storey Steel-Frame College Building.

Superimposed loads :- Roof 60 lb./ft.^{2}
Floors 100 lb./ft.^{2}

Two elevators, each of total weight = 10 tons are supported at mid-span of roof beams Mk. R51. and R61. (Fig. 25)

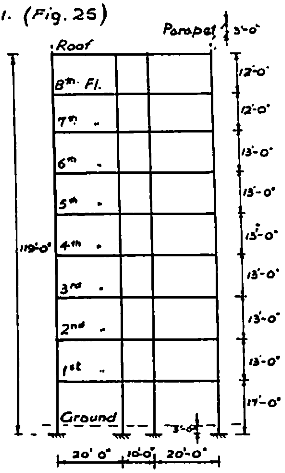


Fig. 24

Uniformly Distributed Load on Roof

Super load	= 60 lb./ft.^{2}
9" slab with 8"×8"×6" deep tiles	= 65 ″
Roof finish, say	10 ″
Ceiling finish, say	5 ″
Total U.D. Load	= 140 lb./ft.^{2}

Main Roof-Beams Mk. R1, R2, R3 etc.

Panel load $2/140^{\#} \times 10' \times \frac{20'}{2} = 28{,}000$ lb.

Beam, self-wt. at say 45 lb./ft.

$45 \times 20' = 900$ ″

Casing, say $\frac{10'' \times 9''}{144} \times 140^{\#} \times 20' = 1{,}750$ ″

Total load on beam = 30,650 lb

= 13·68 tons.

End reaction = 6·84 ″

$$\text{Max. B.M} = \frac{28000 \times 20'}{2240 \times 6'} + \frac{2650 \times 20'}{2240 \times 8}$$

$$= 44 \cdot 56 \text{ ton-ft.}$$

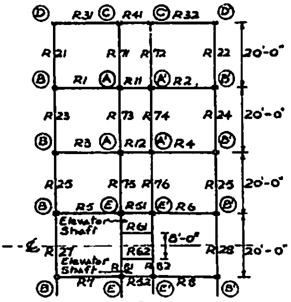
At F_b = 10 tons/in^2.

$$Z \text{ requ'd.} = \frac{44 \cdot 56 \times 12}{10} = 53 \cdot 50 \text{ in}^3$$

Adopt a 15"× 5"× 42 lb. RSJ.

$I = 428 \cdot 49$ in^4 $Z = 57 \cdot 13$ in^3

Use $\frac{3}{4}$" rivets in end-connections (for design of which see Design Sheets B/19 – B/22.)



Roof Plan

Fig. 25

Deflection due to dead load :

(i) Triangular panel load $= \frac{1}{60} \cdot \frac{W.l^3}{E.I} = \frac{1}{60} \times \frac{16{,}000 \times (20' \times 12)^3}{29 \times 10^6 \times 428 \cdot 49} = 0 \cdot 297$ in.

(ii) Beam self-wt. and casing $= \frac{5}{384} \cdot \frac{W.l^3}{E.I} = \frac{5}{384} \times \frac{2{,}650 \times (20' \times 12)^3}{29 \times 10^6 \times 428 \cdot 49} = 0 \cdot 038$ ″

Deflection due to superimposed load

(iii) Triangular panel load $= \frac{1}{60} \cdot \frac{W.l^3}{E.I} = \frac{1}{60} \times \frac{12{,}000 \times (20' \times 12)^3}{29 \times 10^6 \times 428 \cdot 49} = 0 \cdot 223$ ″

Total deflection at mid-span = 0·558 in.

Max. permissible deflection (B.S. 449. Cl. 34.)

$= \frac{\text{span}}{325} = \frac{20' \times 12}{325} = 0 \cdot 739$ ″

B/2.

Main Roof-Beams. Mk. R51. Span 10′-0″

Panel load $2/140^{\#} \times 5 \times \frac{10'}{2}$ = 7,000 lb.

Beam, self-wt. at say 35 lb./ft. $35 \times 10'$ = 350 ·

Casing, say $\frac{10'' \times 4''}{144} \times 140^{\#} \times 10'$ = 390 ·

Elevator reaction $^{*}2(5' \times 2240)$ = 22,400 ·

Total load on beam = 30,140 lb. = 13·46 tons

End reaction = 6·73 ″

$$\text{Max B.M.} = \frac{7{,}000 \times 10}{2240 \times 6} + \frac{740' \times 10'}{2240 \times 8} + \frac{^{*}2(5' \times 10')}{4} = 30{\cdot}61 \text{ ton-ft.}$$

* Reaction and B.M due to elevator load are doubled to provide for impact and vibration affects

$$Z.\ \text{requ'd.} = \frac{30{\cdot}61 \times 12}{10} = 36{\cdot}75 \text{ in}^3$$

Adopt a 10″ × 6″ × 40 lb. RSJ. $I = 204{\cdot}80 \text{ in}^4$ $Z = 40{\cdot}96 \text{ in}^3$

Elevator Roof-Beams Mk. R61.

Total load on beam (as for beam Mk. R51 but without triangular panel load) = 23,140 lb. = 10·80 tons

Adopt a 10″ × 6″ × 40 lb. RSJ as for beam Mk. R51.

Main Roof-Beams Mk. R11, R12, etc. Span 10′-0″

Total load on beam (as for beam Mk. R 51. but without elevator reaction) = 7,740 lb. = 3·46 tons.

End reaction = 1·73 ″

$$\text{Max. B.M.} = \frac{7{,}000 \times 10'}{2240 \times 6} + \frac{740 \times 10'}{2240 \times 8} = 5{\cdot}62 \text{ ton-ft.}$$

$$Z\ \text{requ'd.} = \frac{5{\cdot}62 \times 12}{10} = 6{\cdot}75 \text{ in}^3$$

Thus it appears that a beam of much lighter section than for beam Mk. R51 is called for. Even so it will suit the appearance of the ceilings better and simplify construction if a beam of the same depth – (i.e. 10″) – is adopted. Also additional strength will be of service in centre-bay of frame in resisting non-vertical forces.

Adopt a 10″ × 5″ × 30 lb. RSJ. $I = 146{\cdot}23 \text{ in}^4$ $Z = 29{\cdot}25 \text{ in}^3$

For design of end-connections see Design Sheets B/19 – B/22

End Roof-Beams Mk. R41. Span 20′-0″

Panel load $140 \times 5' \times \frac{10'}{2}$ = 3,500 lb.

Parapet $3{\cdot}5' \times 110^{\#} \times 10'$ = 3,850 ·

Beam, self-wt. and casing, say 740 ·

Total load on beam = 8,090 lb. = 3·62 tons.

End reaction = 1·81 ″

Adopt a 10″ × 5″ × 30 lb. RSJ as for beams Mk R11, R12 etc.

B/3.

Edge Roof-Beams. Mk. R21, R 22. etc. Span 20'-0"

(Similarly for End Roof-Beams Mk. R31. and R32.)

Panel load	$140^{\#} \times 10' \times 20' =$	14,000 lb.	
Parapet	$3{\cdot}5' \times 110^{\#} \times 20' =$	7,700 "	
Beam self-wt. at say 30 lb./ft.	$30 \times 20' =$	600 "	
Casing, say	$\frac{9'' \times 6''}{144} \times 140^{\#} \times 20' =$	1,060 "	
Total load on beam	=	23,350 lb.	= 10·44 tons
End reaction			= 5·22 "

Max B.M. $= \dfrac{14{,}000 \times 20'}{2240 \times 6} + \dfrac{9{,}350 \times 20'}{2240 \times 8} = 31{\cdot}25$ ton-ft.

Z requ'd $= \dfrac{31{\cdot}25 \times 12}{10}$ $= 37{\cdot}50\ \text{in}^3$

Adopt a $12'' \times 5'' \times 32$ lb. RSJ. $I = 221{\cdot}07\ \text{in}^4$ $Z = 36{\cdot}84$ in.

Using $\frac{3}{4}''$ rivets in $2/3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{3}{8}'' \times 9''$-long web-cleat connections:

No of rivets requ'd. in bearing on web $= \dfrac{5{\cdot}22}{0{\cdot}81 \times 0{\cdot}42 \times 12{\cdot}0}$ = 2 (Min.)

No of rivets requ'd. in single-shear $= \dfrac{5{\cdot}22}{\frac{\pi}{4} \times (0{\cdot}81)^2 \times 5{\cdot}0}$ = 4 (say)

(N.B. The end-connections for the End Roof-Beams Mk. R31 and R32. which form part of a transverse frame must also include flange cleats to resist frame moments due to non-vertical forces, See Design Sheet B/19.

Deflection due to dead load:

(i) Triangular panel load $= \dfrac{1}{60} \cdot \dfrac{W.l^3}{E.I} = \dfrac{1}{60} \times \dfrac{8{,}000 \times (20 \times 12)^3}{29 \times 10^6 \times 221{\cdot}07} = 0{\cdot}287$ in.

(ii) Beam self-wt. casing and parapet

$$= \frac{5}{384} \cdot \frac{W.l^3}{E.I} = \frac{5}{384} \times \frac{9{,}350 \times (20 \times 12)^3}{29 \times 10^6 \times 221{\cdot}07} = 0{\cdot}263\ "$$

Deflection due to superimposed load:

(iii) Triangular panel load $= \dfrac{1}{60} \cdot \dfrac{W.l^3}{E.I} = \dfrac{1}{60} \times \dfrac{6{,}000 \times (20 \times 12)^3}{29 \times 10^6 \times 221{\cdot}07} = 0{\cdot}216$ "

Total deflection at mid-span = 0·766 in.

Max. permissible deflection $= \dfrac{\text{span}}{325} = \dfrac{20' \times 12'}{325}$ = 0·739 "

The beam selected will suffice since in fact both dead and live load deflections will be less than those computed due to support contributed by stiffness of parapet, beam casing and wall panel below beam.

(see Fig. 26.)

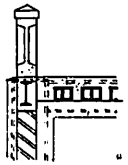


Fig. 26

B/4.

Longitudinal Roof-Beams Mk. R81, R82, etc Span 20'-0"

Panel load (See Fig. 23)
(i) $140^{\#} \times 10' \times \frac{20'}{2}$ = 14.000 lb.
(ii) $140^{\#} \times 5' \times (20' - \frac{10'}{2})$ = 10,500 "

Beam, self-wt. say 55 lb./ft. $55 \times 20'$ = 1,100 "
Casing, say $\frac{10" \times 8"}{144} \times 140^{\#} \times 20'$ = 1,560 "
Reactions from Elevator Roof Beams = 23,140 "
Total load on beam = 50,300 lb. = 22·44 tons.
End reaction = 11·22 "

$$\text{Max. B.M.} = \frac{24{,}500 \times 20'}{2240 \times 6}\ (\text{approx.}) + \frac{2{,}660 \times 20'}{2240 \times 8} + \frac{23140 \times 6'}{2240 \times 2} = 70{\cdot}36 \text{ ton-ft.}$$

$$Z \text{ requ'd.} = \frac{70{\cdot}36 \times 12}{10} = 84{\cdot}4 \text{ in}^3$$

Adopt a 16" × 6" × 62 lb. RSJ.

$I = 725{\cdot}05 \text{ in}^4 \quad Z = 90{\cdot}63 \text{ in}^3$

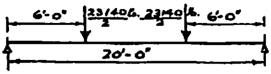


Fig. 27.

Using ¾" rivets in 3½" × 3½" × ⅜" × 1'-0" long web-cleat connections.

No of rivets requ'd in bearing on web (shop rivets.) $= \frac{11{\cdot}22}{0{\cdot}81 \times 0{\cdot}55 \times 12{\cdot}0} = 3$ (Min.)

No of rivets requ'd in single-shear (field rivets.) $= \frac{11{\cdot}22}{\frac{\pi}{4} \times (0{\cdot}81)^2 \times 5{\cdot}0} = 6$ (say).

Longitudinal Roof-Beams. Mk. R71, R72, etc. Span 20'-0"

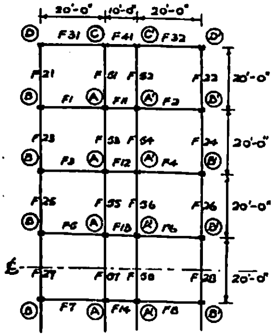
Total load on beam (as for beam Mk. R81, but without reactions from elevator roof beams) = 27,160 lb. = 12·14 tons.
End reaction = 6·07 "
Adopt a 15" × 5" × 42 lb. RSJ as for main roof beams Mk. R1, etc.

Uniformly Distributed Load on Floors.

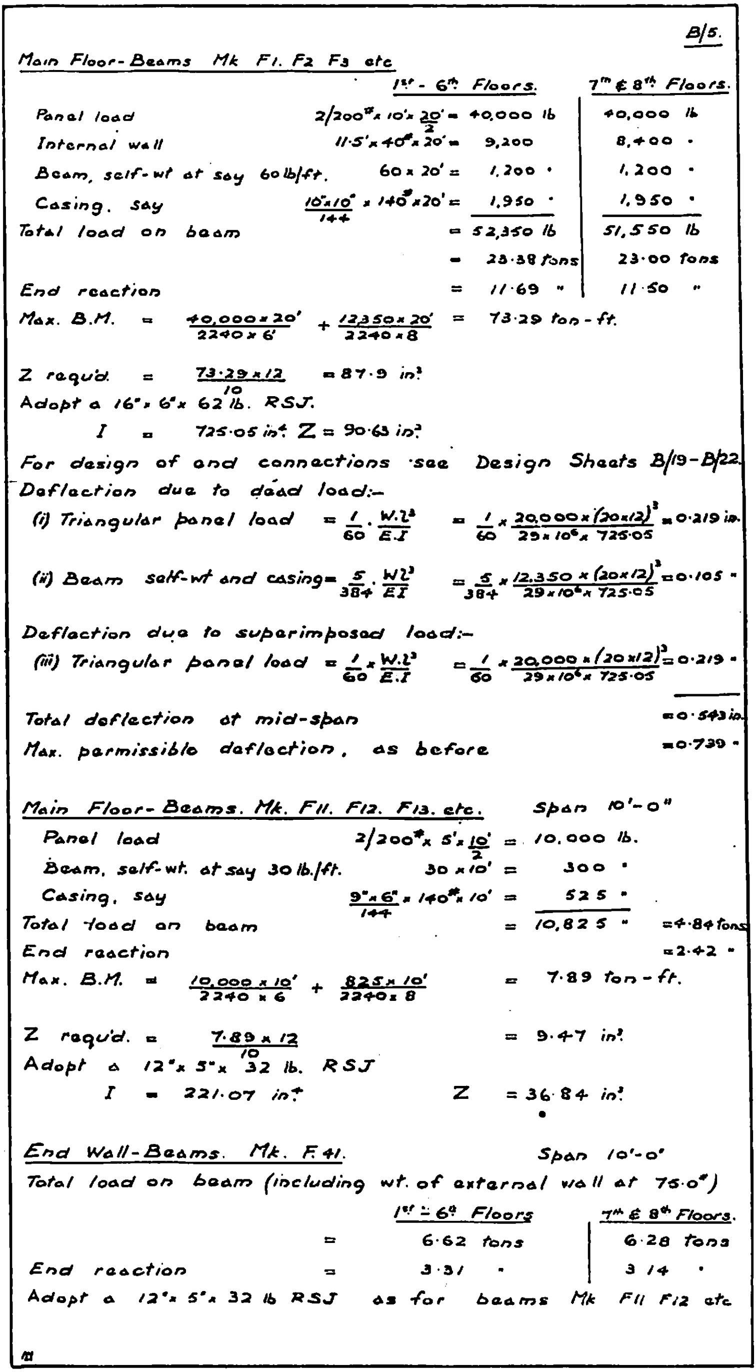
Super. load	= 100 lb/ft²
9" slab with 8" × 8" × 6"-deep tiles	= 65 "
Floor finish, say	10 "
Ceiling finish, say	5 "
Partitions, say	20 "
Total U.D. Load	= 200 lb/ft²

Triangular panel loads accredited to the different beams are indicated in Fig. 23.

The height of the internal and external walls supported by beams at the 7th and 8th floors is 1'-0" less than of those carried by beams at the 1st to 6th floors.



Floor Plan.
Fig. 28.



B/5.

Main Floor-Beams Mk F1. F2 F3 etc

		1st - 6th Floors.	7th & 8th Floors.
Panel load	$2/200^{\#} \times 10' \times \frac{20'}{2}$ =	40,000 lb	40,000 lb
Internal wall	$11{\cdot}5' \times 40^{\#} \times 20'$ =	9,200	8,400 "
Beam, self-wt at say 60 lb/ft.	$60 \times 20'$ =	1,200 "	1,200 "
Casing. say	$\frac{10'' \times 10''}{144} \times 140^{\#} \times 20'$ =	1,950 "	1,950 "
Total load on beam	=	52,350 lb	51,550 lb
	=	23·38 tons	23·00 tons
End reaction	=	11·69 "	11·50 "

$$\text{Max. B.M.} = \frac{40{,}000 \times 20'}{2240 \times 6'} + \frac{12350 \times 20'}{2240 \times 8} = 73{\cdot}29 \text{ ton-ft.}$$

$$Z \text{ requ'd.} = \frac{73{\cdot}29 \times 12}{10} = 87{\cdot}9 \text{ in}^3$$

Adopt a 16" × 6" × 62 lb. RSJ.

$$I = 725{\cdot}05 \text{ in}^4 \quad Z = 90{\cdot}63 \text{ in}^3$$

For design of end connections see Design Sheets B/19-B/22.

Deflection due to dead load:–

(i) Triangular panel load $= \frac{1}{60} \cdot \frac{W.l^3}{E.I} = \frac{1}{60} \times \frac{20{,}000 \times (20 \times 12)^3}{29 \times 10^6 \times 725{\cdot}05} = 0{\cdot}219$ in.

(ii) Beam self-wt and casing $= \frac{5}{384} \cdot \frac{Wl^3}{EI} = \frac{5}{384} \times \frac{12{,}350 \times (20 \times 12)^3}{29 \times 10^6 \times 725{\cdot}05} = 0{\cdot}105$ "

Deflection due to superimposed load:–

(iii) Triangular panel load $= \frac{1}{60} \times \frac{W.l^3}{E.I} = \frac{1}{60} \times \frac{20{,}000 \times (20 \times 12)^3}{29 \times 10^6 \times 725{\cdot}05} = 0{\cdot}219$ "

Total deflection at mid-span = 0·543 in.

Max. permissible deflection, as before = 0·739 "

Main Floor-Beams. Mk. F11. F12. F13. etc. Span 10'-0"

Panel load	$2/200^{\#} \times 5' \times \frac{10'}{2}$ =	10,000 lb.	
Beam, self-wt. at say 30 lb./ft.	$30 \times 10'$ =	300 "	
Casing, say	$\frac{9'' \times 6''}{144} \times 140^{\#} \times 10'$ =	525 "	
Total load on beam	=	10,825 "	= 4·84 tons
End reaction			= 2·42 "

$$\text{Max. B.M.} = \frac{10{,}000 \times 10'}{2240 \times 6} + \frac{825 \times 10'}{2240 \times 8} = 7{\cdot}89 \text{ ton-ft.}$$

$$Z \text{ requ'd.} = \frac{7{\cdot}89 \times 12}{10} = 9{\cdot}47 \text{ in}^3$$

Adopt a 12" × 5" × 32 lb. RSJ

$$I = 221{\cdot}07 \text{ in}^4 \qquad Z = 36{\cdot}84 \text{ in}^3$$

End Wall-Beams. Mk. F.41. Span 10'-0"

Total load on beam (including wt. of external wall at 75·0#)

		1st - 6th Floors	7th & 8th Floors.
	=	6·62 tons	6·28 tons
End reaction	=	3·31 "	3·14 "

Adopt a 12" × 5" × 32 lb RSJ as for beams Mk F11 F12 etc.

B/6.

Side Wall-Beams Mk. F21. F22. F23 etc. Span 20'-0"

(Similarly for End Wall-Beams F31. & F32.)

		1st - 6th Floors	7th & 8th Fls.
Panel load	$200\# \times 10' \times \frac{20'}{2}$ =	20,000 lb.	20000 lb.
External wall	$11.5' \times 75\# \times 20'$ =	17,250 "	15,750 "
Beam, self-wt. at say 45 lb./ft.	$45 \times 20'$ =	900 "	900 "
Casing, say	$\frac{10" \times 9"}{144} \times 140\# \times 20'$ =	1,750 "	1,750 "
Total load on beam	=	39,900 lb.	38,400 lb
	=	17·80 tons.	17·16 tons.
End reaction	=	8·90 "	8·58 "

$$\text{Max. B.M.} = \frac{20{,}000 \times 20'}{2240 \times 6} + \frac{19{,}900 \times 20'}{2240 \times 8'} = 51{\cdot}95 \text{ ton-ft.}$$

$$Z \text{ requ'd.} = \frac{51{\cdot}95 \times 12}{10} = 62{\cdot}4 \text{ in}^3$$

Adopt a 15"×6"×45 lb. RSJ.

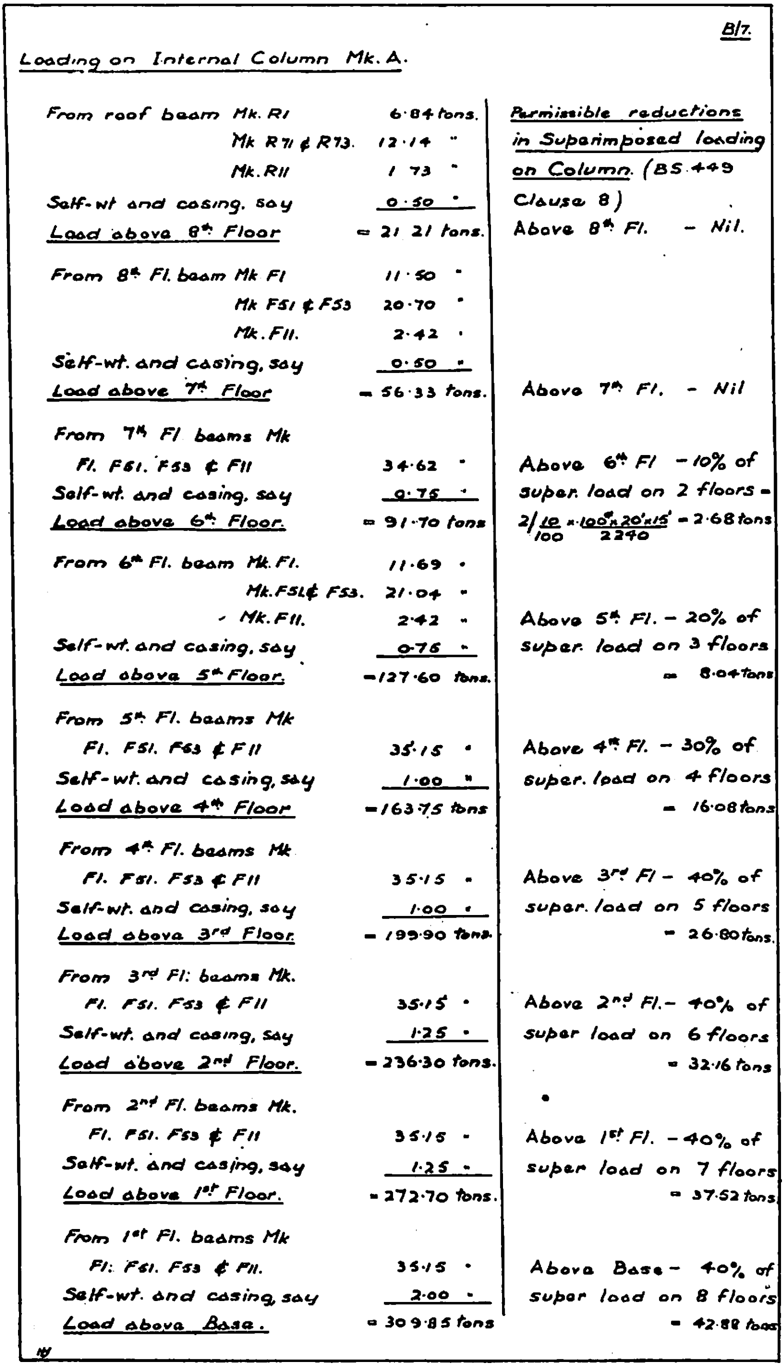
$$I = 491{\cdot}91 \text{ in}^4 \quad Z = 65{\cdot}59 \text{ in}^3$$

Longitudinal Floor-Beams. Mk. F51, F52, F53, etc. Span 20'-0"

			1st - 6th Floors.	7th & 8th Fls.
Panel loads.	(i)	$200\# \times 10' \times \frac{20'}{2}$ =	20,000 lb.	20,000 lb
	(ii)	$200\# \times 5' \times (20' - \frac{10'}{2})$ =	15,000 "	15,000 "
Internal Wall		$11.5' \times 40\# \times 20'$ =	9,200 "	8,400 "
Beam, self-wt. at say 60 lb./ft.		$60 \times 20'$ =	1,200 "	1,200 "
Casing, say		$\frac{10" \times 9"}{144} \times 140\# \times 20'$ =	1,750 "	1,750 "
Total load on beam.		=	47,150 lb.	46,350 lb
		=	21·04 tons	20·70 tons.
End reaction		=	10·52 "	10·35 "

$$\text{Max. B.M.} = \frac{36{,}000 \times 20'}{2240 \times 6} (\text{approx.}) + \frac{12{,}150 \times 20'}{2240 \times 8} = 65{\cdot}71 \text{ ton-ft.}$$

Adopt a 16"×6"×62 lb. RSJ as for main floor beams Mk. F1. F2.
Use ¾" rivets in web-cleat and connections.



B/7.

Loading on Internal Column Mk. A.

From roof beam Mk. R1	6·84 tons.	Permissible reductions
Mk R71 & R73.	12·14 "	in Superimposed loading
Mk. R11	1·73 "	on Column. (BS 449
Self-wt and casing, say	0·50 "	Clause 8)
Load above 8th Floor	= 21·21 tons.	Above 8th Fl. – Nil.
From 8th Fl. beam Mk F1	11·50 "	
Mk F51 & F53	20·70 "	
Mk. F11.	2·42 "	
Self-wt. and casing, say	0·50 "	
Load above 7th Floor	= 56·33 tons.	Above 7th Fl. – Nil
From 7th Fl beams Mk		
F1. F51. F53 & F11	34·62 "	Above 6th Fl – 10% of
Self-wt. and casing, say	0·75 "	super. load on 2 floors =
Load above 6th Floor.	= 91·70 tons	$2/\frac{10}{100} \times \frac{100^{\#} \times 20' \times 15'}{2240}$ = 2·68 tons
From 6th Fl. beam Mk. F1.	11·69 "	
Mk. F51 & F53.	21·04 "	
Mk. F11.	2·42 "	Above 5th Fl. – 20% of
Self-wt. and casing, say	0·76 "	super. load on 3 floors
Load above 5th Floor.	= 127·60 tons.	= 8·04 tons
From 5th Fl. beams Mk		
F1. F51. F53 & F11	35·15 "	Above 4th Fl. – 30% of
Self-wt. and casing, say	1·00 "	super. load on 4 floors
Load above 4th Floor	= 163·75 tons	= 16·08 tons
From 4th Fl. beams Mk		
F1. F51. F53 & F11	35·15 "	Above 3rd Fl – 40% of
Self-wt. and casing, say	1·00 "	super. load on 5 floors
Load above 3rd Floor.	= 199·90 tons.	= 26·80 tons.
From 3rd Fl: beams Mk.		
F1. F51. F53 & F11	35·15 "	Above 2nd Fl. – 40% of
Self-wt. and casing, say	1·25 "	super load on 6 floors
Load above 2nd Floor.	= 236·30 tons.	= 32·16 tons
From 2nd Fl. beams Mk.		
F1. F51. F53 & F11	35·15 "	Above 1st Fl. – 40% of
Self-wt. and casing, say	1·25 "	super load on 7 floors
Load above 1st Floor.	= 272·70 tons.	= 37·52 tons
From 1st Fl. beams Mk		
F1: F51. F53 & F11.	35·15 "	Above Base – 40% of
Self-wt. and casing, say	2·00 "	super load on 8 floors
Load above Base.	= 309·85 tons	= 42·88 tons

B/8.

Internal Column Mk. A

Assume a 14"× 8"× 70 lb RSJ with 2/14"× 1⅛" plates in the 1st storey tapering to a plain 14"× 6"× 46 lb RSJ in the 9th storey and check section at position of splices above 7th, 5th, 3rd & 1st Floors (See Fig 29, Design Sheet B/11.)

Above 7th Floor

Assume a 14"× 6"× 46 lb RSJ above 7th Fl and a 14"× 6"× 46 lb RSJ with 2/12"× ⅜" plates below 7th Fl

For 8th storey columns $L = 144"$ $l = 144 \times 0{\cdot}7 = 101"$

$A = 13\ 59\ in^2$

$I_{xx} = 442\ 57\ in^4$

$Z_{xx} = 63\ 22\ in^3$

$r_{xx} = 5\ 71\ in$ $r_{yy} = 1{\cdot}26\ in.$

With concrete casing:-   Min. $r_{yy} = 0{\cdot}2(b+4) = 2\ 00"$

$\frac{l}{r} = \frac{101}{2{\cdot}0} = 51{\cdot}5$ for which $F_a = 6\ 54\ tons/in^2$

Direct load = 56·33 tons

and $f_a = \frac{56{\cdot}33}{13{\cdot}59} = 4{\cdot}15\ tons/in^2$

Eccentricities of beam reactions at 7th Fl.:-

$e_{xx} = 7" + \frac{3}{8}" + 1"(say) = 8{\cdot}38"$ $e_{yy} = 1"(say)$

For 8th storey columns $\frac{I_{xx}}{L} = \frac{442\ 57}{144} = 3{\cdot}07$

For 7th storey columns $\frac{I_{xx}}{L} = \frac{907{\cdot}57}{156} = 5{\cdot}82$ $\frac{5{\cdot}82}{3{\cdot}07} > 1{\cdot}5$

Bending moment in 8th storey columns at 7th Fl.

$M_{xx} = \frac{3{\cdot}07}{8{\cdot}95}(11{\cdot}50-2{\cdot}42).8{\cdot}38" = 26{\cdot}1$ ton-ins. $M_{yy} = 0$

$f_{bc} = \frac{26{\cdot}1}{63{\cdot}22} = 0\ 41\ tons/in^2$ $F_{bc} = 10{\cdot}0\ tons/in^2$

(BS. 449. Cl. 19d)

and $\frac{f_a}{F_a} + \frac{f_{bc}}{F_{bc}} = \frac{4{\cdot}15}{6{\cdot}54} + \frac{0{\cdot}41}{10{\cdot}0} = 0{\cdot}676$ - accepted!

Above 5th Floor.

Assume a 14"× 6"× 46 lb RSJ with 2/12"× ⅜" plates above 5th Fl and a 14"× 6"× 46 lb RSJ with 2/12"× ¾" plates below 5th Fl.

For 6th storey columns $L = 156"$. $l = 156 \times 0{\cdot}7 = 109"$

$A_{xx} = 13\ 59 + 9\ 00 = 22\ 59\ in^2$

$I_{xx} = 442\ 57 + 465{\cdot}00 = 907\ 57\ in^4$

$Z_{xx} = \frac{907{\cdot}57 \times 2}{14{\cdot}75} = 123\ 05\ in^3$

With concrete casing:-   Min $r_{yy} = 0{\cdot}2(b+4) = 3{\cdot}2"$

$\frac{l}{r} = \frac{109}{3\ 2} = 34$ for which $F_a = 7{\cdot}35\ tons/in^2$

Direct load = 127·60 − 8·04 = 119·56 tons

and $f_a = \frac{119{\cdot}56}{22{\cdot}59} = 5\ 30\ tons/in^2$

Eccentricity of beam reactions at 5th Fl -

$e_{xx} = 7" + \frac{3}{4}" + 1"(say) = 8{\cdot}75"$

B/9.

Internal Column Mk. A (Contd.)

Above 5th Floor (Contd.).

For 6th storey columns $\frac{I_{xx}}{L} = \frac{907.57}{156} = 5.81$
For 5th storey columns $\frac{I_{xx}}{L} = \frac{1422.49}{156} = 9.13$
$\left.\right\} \frac{9.13}{5.81} > 1.50$

Bending moment in 6th storey columns at 5th Fl. :-

$M_{xx} = \frac{5.81}{14.94}(11.69 - 2.42).8.75 = 31.5$ ton-ins. $M_{yy} = 0$

$f_{bc} = \frac{31.5}{123.05} = 0.26$ tons/in^2.

and $\frac{f_a}{F_a} + \frac{f_{bc}}{F_{bc}} = \frac{5.30}{7.35} + \frac{0.26}{10.0} = 0.748$. – accepted!.

Above 3rd Floor.

Assume a 14" × 6" × 46 lb. RSJ with 2/12" × ¾" plates above 3rd Fl.
and a 14" × 8" × 70 lb. RSJ with 2/14" × ¾" plates below 3rd Fl.

For 4th storey columns $L = 156"$. $l = 156" \times 0.7 = 109"$.

$A = 13.59 + 18.00 = 31.59$ in^2.

$I_{xx} = 442.57 + 979.92 = 1422.49$ in^4.

$Z_{xx} = \frac{1422.49 \times 2}{15.50} = 183.50$ in^3.

With concrete casing :- Min. $r_{yy} = 0.2(b+4) = 0.32"$

$\frac{l}{r} = \frac{109}{3.2} = 34$ for which $F_a = 7.35$ tons/in^2.

Direct load $= 199.90 - 26.80 = 173.10$ tons.

and $f_a = \frac{173.10}{31.59} = 5.49$ tons/in^2.

$e_{xx} = 8.75"$ as before.

For 4th storey columns $\frac{I_{xx}}{L} = \frac{1422.49}{156} = 9.13$
For 3rd storey columns $\frac{I_{xx}}{L} = \frac{1848.82}{156} = 11.85$
$\left.\right\} \frac{11.85}{9.13} < 1.50$.

Bending moment in 4th storey columns at 3rd Fl..

$M_{xx} = \frac{1}{2}(11.69 - 2.42).8.75$ 40.60 ton-ins.

$f_{bc} = \frac{40.60}{183.50} = 0.22$ tons/in^2.

and $\frac{f_a}{F_a} + \frac{f_{bc}}{F_{bc}} = \frac{5.49}{7.35} + \frac{0.22}{10.0} = 0.769$ – accepted!.

Above 1st Floor.

Assume a 14" × 8" × 70 lb. RSJ with 2/14" × ¾" plates above 1st Fl.
and a 14" × 8" × 70 lb. RSJ with 2/14" × 1⅛" plates below 1st Fl.

For 2nd storey columns $L = 156"$ $l = 156 \times 0.7 = 109"$

$A = 20.59 + 21.00 = 41.59$ in^2.

$I_{xx} = 705.58 + 1143.24 = 1848.82$ in^4.

$Z_{xx} = \frac{1848.82}{15.50} = 238.50$ in^3.

With concrete casing Min. $r_{yy} = 0.2(b+4) = 3.6"$

$\frac{l}{r} = \frac{109}{3.2} = 30$. for which $F_a = 7.54$ tons/in^2

Direct load $272.70 - 37.52 = 235.18$ tons

and $f_a = \frac{235.18}{41.59} = 5.66$ tons/in^2.

B/10.

Internal Column Mk. A. (Cont'd.)

Above 1st Floor. (Cont'd.)

Eccentricity of beam reactions at 1st Fl. :-

$e_{xx} = 7" + 1\frac{1}{8}" + 1"(\text{say}) = 9{\cdot}12"$ $\qquad e_{yy} - 1"(\text{say})$

$$\left.\begin{array}{ll}\text{For 2nd storey columns } \dfrac{I_{xx}}{L} = \dfrac{1848{\cdot}82}{156} & = 11{\cdot}85 \\ \text{For 1st storey columns } \dfrac{I_{xx}}{L} = \dfrac{2510{\cdot}46}{204} & = 12{\cdot}32\end{array}\right\} \frac{12{\cdot}32}{11{\cdot}85} < 1{\cdot}50$$

Bending moment in 2nd storey columns at 1st Fl. :-

$$M_{xx} = \tfrac{1}{2}.(11{\cdot}69 - 2{\cdot}42).9{\cdot}12 = 42{\cdot}30 \text{ ton-ins.}$$

$$f_{bc} = \frac{42{\cdot}30}{238{\cdot}50} = 0{\cdot}18 \text{ tons/in}^2.$$

$$\text{and } \frac{f_a}{F_a} + \frac{f_{bc}}{F_{bc}} = \frac{5{\cdot}66}{7{\cdot}54} + \frac{0{\cdot}18}{10{\cdot}0} = 0{\cdot}770 \text{ – accepted!.}$$

Base to 1st Floor.

Assume a 14" x 8" x 70 lb. RSJ with 2/14" x $1\frac{1}{8}"$ plates.

$$A = 20{\cdot}59 + 31{\cdot}50 = 52{\cdot}09 \text{ in}^2$$

$$I_{xx} = 705{\cdot}58 + 1804{\cdot}88 = 2510{\cdot}46 \text{ in}^4.$$

$$Z = \frac{2510{\cdot}46 \times 2}{16{\cdot}25} = 309{\cdot}50 \text{ in}^3$$

For 1st storey columns $L = 204"$ $\quad l = 204 \times 0{\cdot}75 = 153"$

With concrete casing :- $\quad$ Min. $r_{yy} = 0{\cdot}2(b+4) = 3{\cdot}6"$

$$\frac{l}{r} = \frac{153}{3{\cdot}6} = 42{\cdot}5 \text{ for which } F_a = 6{\cdot}94 \text{ tons/in}^2$$

Direct load = $309{\cdot}85 - 42{\cdot}88 = 266{\cdot}97$ tons.

$$\text{and } f_a = \frac{266{\cdot}97}{52{\cdot}09} = 5{\cdot}13 \text{ tons/in}^2.$$

Bending moment in 1st storey columns at 1st Fl. :-

$$M_{xx} = 42{\cdot}30 \text{ ton-ins.}$$

$$f_{bc} = \frac{42{\cdot}30}{309{\cdot}50} = 0{\cdot}14 \text{ tons/in}^2.$$

$$\text{and } \frac{f_a}{F_a} + \frac{f_{bc}}{F_{bc}} = \frac{5{\cdot}13}{6{\cdot}94} + \frac{0{\cdot}14}{10{\cdot}0} = 0{\cdot}754. \text{ – accepted!}$$

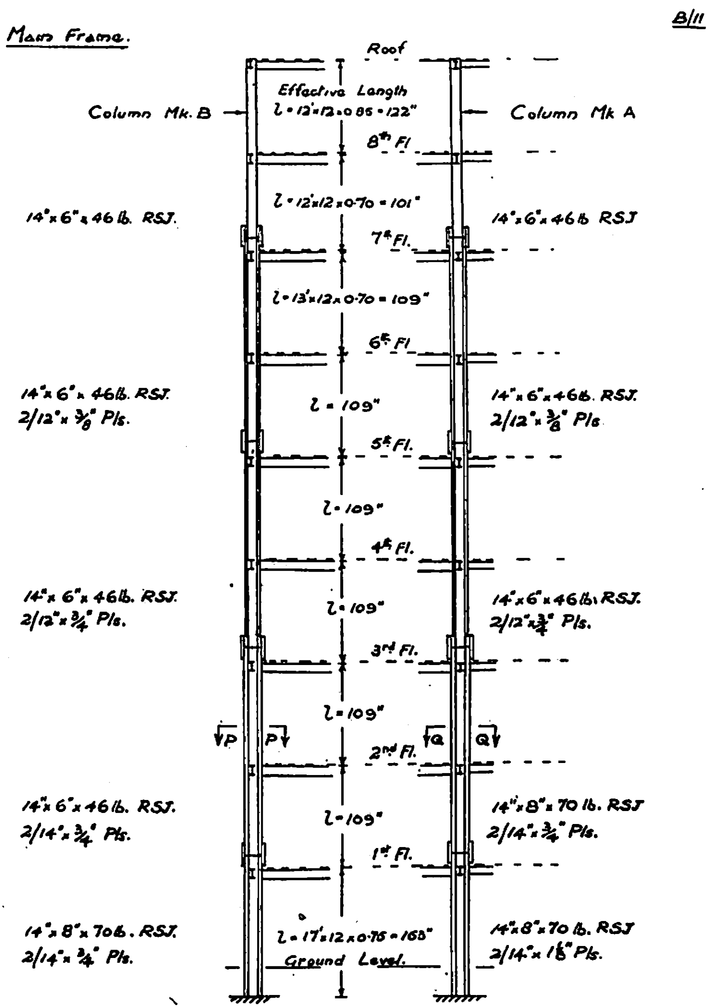
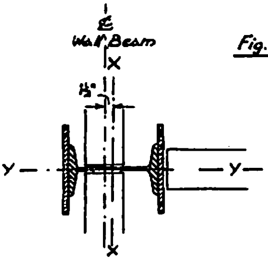
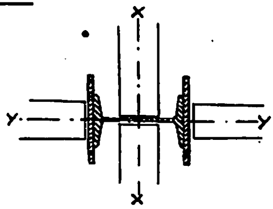


Fig. 29 (a).



Section P-P.
Fig 29(b)



Section Q-Q
Fig. 29 (c)

B/12

Loading on External Column Mk B.

From roof beam Mk R1.	6·84 tons	Permissible reductions
Mk R21 & R23	10·44	in Superimposed
Self-wt. and casing; say	0·50	loading on Column:-
Load above 8th Floor.	= 17·78 tons	Above 8th Fl – Nil.
From 8th Fl beam Mk. F1.	11·50	
Mk. F21. & F23.	17·16	
Self-wt. and casing, say	0·50	
Load above 7th Floor.	= 46·94 tons	Above 7th Fl. – Nil.
From 7th Fl. beam Mk. F1	11·50	Above 6th Fl. –10% of
Mk. F21 & F23.	17·16	Super. load on 2 floors
Self-wt. and casing, say	0·67	= $2/\frac{10}{100} \times 100^{\#} \times 20' \times 10'$
Load above 6th Floor.	= 76·27 tons	= 1·79 tons
From 6th Fl. beam Mk. F1.	11·69	
Mk. F21 & F23.	17·80	Above 5th Fl. –20% of
Self-wt. and casing, say	0·67	super. load on 3 floors
Load above 5th Floor.	= 106·43 tons	= 5·37 tons
From 5th Fl. beam Mk. F1.	11·69	
Mk. F21 & F23.	17·80	Above 4th Fl. – 30% of
Self-wt. and casing, say	0·75	super. load on 4 floors
Load above 4th Floor.	= 136·67 tons	= 10·74 tons
From 4th Fl. beam Mk. F1.	11·69	
Mk. F21 & F23.	17·80	Above 3rd Fl. – 40% of
Self-wt. and casing, say	0·75	super. load on 5 floors
Load above 3rd Floor.	= 166·91 tons.	= 17·90 tons
From 3rd Fl. beam Mk. F1.	11·69	
Mk. F21. & F23.	17·80	Above 2nd Fl. – 40% of
Self-wt and casing, say	1·00	super. load on 6 floors
Load above 2nd Floor.	= 197·40 tons	= 21·48 tons
From 2nd Fl. beam Mk. F1.	11·69	
Mk. F21. & F23	17·80	Above 1st Fl. –40% of
Self-wt. and casing, say	1·00	super. load on 7 floors
Load above 1st Floor.	= 227·89 tons.	= 25·06 tons
From 1st Fl. beam Mk. F1.	11·69	
Mk. F21 & F23	17·80	Above Base – 40% of
Self-wt. and casing, say	1·50	super. load on 8 floors
Load above Base	= 258·88 tons.	= 28·64 tons

B/13.

External Column. Mk. B.

Assume a 14"×8"×70 lb. RSJ with 2/14"×¾" plates in the 1st storey, tapering to a plain 14"×6"×46 lb. RSJ in the 9th storey.

Above 3rd Floor

Loads at each floor level are less than for corresponding level of internal column Mk. A. and it is considered that the same column sections as in the 4th to 9th storeys of the internal column are suitable here.

Above 1st Floor.

Assume a 14"×6"×46 lb. RSJ with 2/14"×¾" plates above 1st Fl. and a 14"×8"×70 lb. RSJ with 2/14"×¾" plates below 1st Fl.

For 2nd storey columns $L = 156''$. $l = 156 \times 0{\cdot}7 = 109''$

$$A = 13{\cdot}59 + 21{\cdot}00 = 34{\cdot}59 \text{ in}^2$$

$$I_{xx} = 442{\cdot}57 + 1143{\cdot}24 = 1585{\cdot}81 \text{ in}^4$$

$$Z_{xx} = \frac{1585{\cdot}81 \times 2}{15{\cdot}50} = 204{\cdot}5 \text{ in}^3$$

With concrete casing :- Min. $r_{yy} = 0{\cdot}2(b+4) = 3{\cdot}6''$

$$\frac{l}{r} = \frac{109}{3{\cdot}6} = 30 \quad \text{for which} \quad F_a = 7{\cdot}54 \text{ tons/in}^2$$

Direct load $= 227{\cdot}89 - 25{\cdot}06 = 202{\cdot}83$ tons.

$$\text{and } f_a = \frac{202{\cdot}83}{34{\cdot}59} = 5{\cdot}86 \text{ tons/in}^2.$$

Roof beams Mk. R21. R23 etc. and Floor beams Mk. F21. F23. etc. are connected to web of external columns at 1½" "off-centre" (see Fig. 29.) to reduce the bending-moment M_{xx} on the column.

Eccentricities of beam reactions at 1st Fl. :-

$$e_{xx} = 7'' + \tfrac{3}{4}'' + 1''(\text{say}) = 8{\cdot}75'' \text{ for main floor beam Mk. F.}$$

$$\text{and } e_{xx} = 1{\cdot}50'' \text{ for beams Mk. F21. \& F23.}$$

$$M_{xx} = \tfrac{1}{2}(11{\cdot}69 \times 8{\cdot}75 - 17{\cdot}80 \times 1{\cdot}5) = 37{\cdot}8 \text{ ton-ins.}$$

$$f_{bc} = \frac{37{\cdot}8}{204{\cdot}5} = 0{\cdot}19 \text{ tons/in}^2.$$

$$\text{and } \frac{f_a}{F_a} + \frac{f_{bc}}{F_{bc}} = \frac{5{\cdot}86}{7{\cdot}54} + \frac{0{\cdot}19}{10{\cdot}0} = 0{\cdot}797 \text{ -- accepted!}$$

Above Base

Assume a 14"×8"×70 lb. RSJ with 2/14"×¾" plates.

$$A = 20{\cdot}59 + 21{\cdot}00 = 41{\cdot}59 \text{ in}^2.$$

$$I_{xx} = 05{\cdot}58 + 1143{\cdot}24 = 1848{\cdot}82 \text{ in}^4.$$

$$Z_{xx} = \frac{1848{\cdot}82 \times 2}{15{\cdot}50} \quad 238{\cdot}70 \text{ in}^3.$$

$$\frac{l}{r} = 42{\cdot}5 \text{ as before,} \quad \text{for which} \quad F_a = 6{\cdot}94 \text{ tons/in}^2$$

Direct load $= 258{\cdot}88 - 28{\cdot}64 = 230{\cdot}24$ tons

$$\text{and } f_a = \frac{230{\cdot}24}{41{\cdot}59} = 5{\cdot}60 \text{ tons/in}^2.$$

$$M_{xx} = 37{\cdot}80 \text{ ton-ins.}$$

$$f_{bc} = \frac{37{\cdot}80}{238{\cdot}70} = 0{\cdot}16 \text{ tons/in}^2.$$

$$\text{and } \frac{f_a}{F_a} + \frac{f_{bc}}{F_{bc}} = \frac{5{\cdot}60}{6{\cdot}94} + \frac{0{\cdot}16}{10{\cdot}0} = 0{\cdot}824 \text{ -- accepted}$$

B/14

Loading on Corner Column. Mk. D.

From Roof beam Mk. R21.	5·22 tons.	Permissible reductions in
Mk. R31.	5·22 "	Superimposed loading
Self-wt. and casing, say	0·38 "	on Column.
Load above 8th Floor	= 10·82 tons.	Above 8th Fl — Nil.
From 8th Fl. beam Mk. F21.	8·58 "	
Mk. F31.	8·58 "	
Self-wt. and casing, say	0·38 "	
Load above 7th Floor	= 28·36 tons.	Above 7th Fl. — Nil.
From 7th Fl. beam Mk. F21.	8·58 "	Above 6th Fl. — 10% of
Mk. F31.	8·58 "	super. load on 2 floors
Self-wt. and casing, say	0·50 "	$= 2/\frac{10'}{100} \times \frac{100 \times 10' \times 10'}{2240}$
Load above 6th Floor.	= 46·02 tons.	= 0·89 tons.
From 6th Fl. beam Mk. F21.	8·90 "	
Mk. F31.	8·90 "	Above 5th Fl. — 20% of
Self-wt. and casing, say	0·50 "	super. load on 3 floors
Load above 5th Floor	= 64·32 tons.	= 2·68 tons.
From 5th Fl. beam Mk. F21.	8·90 "	
Mk. F31.	8·90 "	Above 4th Fl. — 30% of
Self-wt. and casing, say	0·75 "	super. load on 4 floors
Load above 4th Floor.	= 82·87 tons.	= 5·37 tons.
From 4th Fl. beam Mk. F21.	8·90 "	
Mk. F31.	8·90 "	Above 3rd Fl. — 40% of
Self-wt. and casing, say	0·75 "	super. load on 5 floors
Load above 3rd Floor	= 101·42 tons.	= 8·95 tons.
From 3rd Fl. beam Mk. F21.	8·90 "	
Mk. F31	8·90 "	Above 2nd Fl. — 40% of
Self-wt. and casing, say	1·00 "	super. load on 6 floors
Load above 2nd Floor	= 120·22 tons.	= 10·74 tons
From 2nd Fl. beam Mk. F21.	8·90 "	
Mk. F31.	8·90 "	Above 1st Fl. — 40% of
Self-wt. and casing, say	1·00 "	super. load on 7 floors
Load above 1st Floor	= 139·02 tons.	= 12·53 tons
From 1st Fl. beam Mk. F21.	8·90 "	
Mk. F31.	8·90 "	Above Base — 40% of
Self-wt. and casing, say	1·33 "	super. load on 8 floors
Load above Base	= 158·15 tons.	= 14·32 tons.

B/15.

Corner Column Mk. D

Assume a 12" × 6" × 44 lb RSJ with 2/12" × ¾" plates in the 1st storey, tapering to a plain 12" × 6" × 44 lb RSJ in the 9th storey

Above 5th Floor.

Assume a plain 12" × 6" × 44 lb RSJ above 5th Fl. and a 12" × 6" × 44 lb. RSJ with 2/12" × ⅜" plates below 5th Fl.

For 6th storey columns $L = 156"$ $l = 156 \times 0.7 = 109"$

$A = 13.00 \text{ in}^2$

$I_{xx} = 316.76 \text{ in}^4$ $\qquad I_{yy} = 22.12 \text{ in}^4$

$Z_{xx} = 62.79 \text{ in}^3$ $\qquad Z_{yy} = 7.37 \text{ in}^3$

With concrete casing :- Min. $r_{yy} = 0.2(b+4) = 2.0"$

$\frac{l}{r} = \frac{101}{2.0} = 50.5$ for which $F_a = 6.54 \text{ tons/in}^2$

Direct load $= 64.32 - 2.68 = 61.64$ tons

and $f_a = \frac{61.64}{13.00} = 4.75 \text{ tons/in}^2$

Reactions from both beams Mk. F21. (Mk. R21. at roof) and Mk. F31. (Mk. R31.) framing into column at each floor are eccentric about the X-X axis (see Fig. 29.) and the reaction from beam Mk. F21 is also eccentric about the Y-Y axis.

Eccentricities of reaction from beam Mk. F21. at 5th Fl. :-

$e_{xx} = 1.5"$ $\qquad e_{yy} = 1.0"$ (say)

Eccentricities of reaction from beam Mk. F31. at 5th Fl. :-

$e_{xx} = 6" + ⅜" + 1"$ (say) $= 7.38"$. $\quad e_{yy} = 0$

$$\left.\begin{array}{ll} \text{For 6th storey columns } \frac{I_{xx}}{L} = \frac{316.76}{156} & = 2.03 \\ \text{For 5th storey columns } \frac{I_{xx}}{L} = \frac{661.40}{156} & = 4.23 \end{array}\right\} \frac{4.23}{2.03} > 1.50$$

$$\left.\begin{array}{ll} \text{For 6th storey columns } \frac{I_{yy}}{L} = \frac{22.12}{156} & = 0.14 \\ \text{For 5th storey columns } \frac{I_{yy}}{L} = \frac{130.12}{156} & = 0.83 \end{array}\right\} \frac{0.83}{0.14} > 1.50$$

Bending moments in 6th storey columns at 5th Fl. :-

$M_{xx} = \frac{2.03}{6.26}(8.90 \times 5.88") = 17.0$ ton-ins. $\quad M_{yy} = \frac{0.14}{9.57}(8.90 \times 1.0)$

$= 1.28$ ton-ins

$f_{bc.xx} = \frac{17.0}{52.79} = 0.32 \text{ tons/in}^2$. $\quad f_{bc\,yy} = \frac{1.28}{7.37} = 0.17 \text{ tons/in}^2$

$f_{bc} = f_{bc\,xx} + f_{bc\,yy} = 0.32 + 0.17 = 0.49 \text{ tons/in}^2$

and $\frac{f_a}{F_a} + \frac{f_{bc}}{F_{bc}} = \frac{4.76}{6.54} + \frac{0.49}{10.0} = 0.787$ - accepted'

B/16.

Corner Column Mk. D. (Contd)

Above 1st Floor.

Adopt a 12" × 6" RSJ with 2/12" × 3/8" plates in the 2nd to 5th storeys

$A = 13{\cdot}00 + 9{\cdot}00 = 22{\cdot}00 \text{ in}^2$

$I_{xx} = 316{\cdot}76 + 344{\cdot}64 = 661{\cdot}40 \text{ in}^4 \quad I_{yy} = 22{\cdot}12 + 108{\cdot}00 = 130{\cdot}12 \text{ in}^4$

Above Base

Assume a 12" × 6" × 44 lb RSJ with 2/12" × 3/4" plates in 1st storey.

For 1st storey columns $L = 204" \quad l = 204 \times 0{\cdot}75 = 153"$

$A = 13{\cdot}00 + 18{\cdot}00 = 31{\cdot}00 \text{ in}^2$

$I_{xx} = 316{\cdot}76 + 732{\cdot}36 = 1049{\cdot}12 \text{ in}^4 \quad I_{yy} = 22{\cdot}12 + 216{\cdot}00 = 238{\cdot}12 \text{ in}^4$

$Z_{xx} = \dfrac{1049{\cdot}12 \times 2}{13{\cdot}50} = 155{\cdot}50 \text{ in}^3 \quad Z_{yy} = \dfrac{238{\cdot}12 \times 2}{12{\cdot}00} = 39{\cdot}69 \text{ in}^3$

With concrete casing:- $\quad \text{Min } r_{yy} = 0{\cdot}2(b+4) = 3{\cdot}2"$

$\dfrac{l}{r} = \dfrac{153}{3{\cdot}2} = 48$ for which $F_a = 6{\cdot}67 \text{ tons/in}^2$

Direct load $= 158{\cdot}15 - 14{\cdot}32 = 143{\cdot}83$ tons.

and $f_a = \dfrac{143{\cdot}83}{31{\cdot}0} = 4{\cdot}64 \text{ tons/in}^2$

Eccentricities of reactions from beam Mk. F21. at 1st Fl. :-

$e_{xx} = 1{\cdot}5" \qquad e_{yy} = 1{\cdot}0"$ (say)

Eccentricities of reactions from beam Mk. F31. at 1st Fl. :-

$e_{xx} = 6" + \tfrac{3}{4}" + 1"$ (say) $= 7{\cdot}75" \quad e_{yy} = 0$

$$\left.\begin{array}{lll} \text{For 2nd storey columns} & \dfrac{I_{xx}}{L} = \dfrac{661{\cdot}40}{156} & = 4{\cdot}24 \\ \text{" 1st " "} & \dfrac{I_{xx}}{L} = \dfrac{1049{\cdot}12}{204} & = 5{\cdot}14 \end{array}\right\} \dfrac{5{\cdot}14}{4{\cdot}24} < 1{\cdot}50$$

$$\left.\begin{array}{lll} \text{" 2nd " "} & \dfrac{I_{yy}}{L} = \dfrac{130{\cdot}12}{156} & = 0{\cdot}83 \\ \text{" 1st " "} & \dfrac{I_{yy}}{L} = \dfrac{238{\cdot}12}{204} & = 1{\cdot}17 \end{array}\right\} \dfrac{1{\cdot}17}{0{\cdot}83} < 1{\cdot}50$$

Bending moments in 1st storey columns at 1st Fl :-

$M_{xx} = \tfrac{1}{2}(8{\cdot}90 \times 6{\cdot}25") = 27{\cdot}8 \text{ ton-ins.} \quad M_{yy} = \tfrac{1}{2}(8{\cdot}90 \times 1{\cdot}0) = 4{\cdot}45 \text{ ton-ins.}$

$f_{bc.xx} = \dfrac{27{\cdot}8}{155{\cdot}50} = 0{\cdot}18 \text{ tons/in}^2 \quad f_{bc.yy} = \dfrac{4{\cdot}45}{39{\cdot}69} = 0{\cdot}11 \text{ tons/in}^2$

$f_{bc} = f_{bc\,xx} + f_{bc.yy} = 0{\cdot}18 + 0{\cdot}11 = 0{\cdot}29 \text{ tons/in}^2$

and $\dfrac{f_a}{F_a} + \dfrac{f_{bc}}{F_{bc}} = \dfrac{4{\cdot}64}{6{\cdot}67} + \dfrac{0{\cdot}29}{10{\cdot}0} = 0{\cdot}726$ – accepted!

HY

levels of the splices only. In this case, as in Example (A) the columns are solidly encased in concrete, and their design accords with Clauses 18 (*b*) and 22 (*a*) of B.S. 449. It is considered that the external appearance of the building of Design Example (B) benefits by arranging for the side wall-beams Mk. F21, F22, etc., to connect "off centre" at the webs of the external columns (Fig. 29 (*b*)), since the casing to the columns projects beyond the line of the wall of the building less when this is the case than if the centre-line of the wall-beams coincided with the major axis of the column-section. The amount of this eccentricity of the wall-beams connection about the major axis of the external column-section is limited to 1½″ to avoid cutting away the flanges of the beam and to simplify the connection to the web of the corner column Mk. D for which the basic column section is a 12″ × 8″ R.S.J. A further advantage of this eccentricity is that it occurs on the opposite side of the major axis of the section of the external column to the main floor beam connection, thus reducing the bending moment on the column at each floor level.

Wind Stress Analysis.—From Part I it is seen that the effects of wind forces on a building-frame must be considered (*a*) on the frame as a whole and (*b*) on the individual members and connections of the frame.

The height of the building of Design Example (B) above ground (119 ft.) is less than three times the effective width (50 ft.), and since the concrete floors and walls of the building, together with the concrete casing to the main frame members, result in a considerable stiffening of the building, it may be held that the conditions of Clause 12 (*a*) of B.S. 449 (1948) are complied with, and that the general stability calculations of Design Sheet C/1 are unnecessary. However, since it is possible to estimate both the overturning moment due to wind forces, and the stability moment due to the dead load of the building, whereas the stiffening effect of walls, floors and casing cannot be assessed with the same degree of certainty, it is considered that the computations of Design Sheet C/1 should be carried out for the building under consideration to ensure that such further requirements as that of Clause 30 (*b*) of the British Standard are met.

Several methods of stress analysis for multi-storey building frames under the influence of wind forces have been devised and generally, as with the methods of analysis for vertical loading, the more exact the method the more cumbersome it is to employ. An approximate method commonly employed—the so-called "portal method"—is adopted here on the basis of a number of assumptions which are made with a view to simplifying and reducing the design computations, viz. :—

C/1.

Stability of Frame Subjected to Wind Forces.

Wind Forces (B S 449 Cl.12.)

P_1 = $0{\cdot}0074^{\#} \times 20' \times 92'$ = 13·62 tons

P_2 = $0{\cdot}0074 \times 20' \times 25'$ = 3 70 "

P_3 = $0\ 5 \times P_2$ = 1 85 "

Overturning Moment due to wind forces about base of external leeward column L

= $P_1 \times 76{\cdot}0' + P_2 \times 37{\cdot}5' + P_3 \times 12\ 5'$

= $13{\cdot}62 \times 76{\cdot}0' + 3{\cdot}70 \times 37{\cdot}5' + 1{\cdot}85 \times 12{\cdot}5'$

= 1035·12 + 138 75 + 23·12

= 1197·0 ton-ft

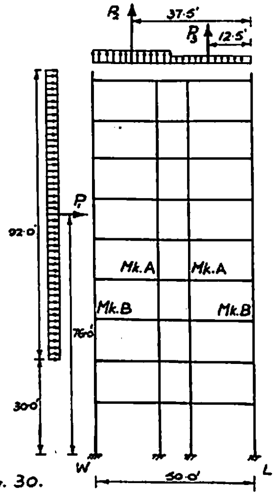


Fig. 30.

Dead Load on Columns.

Internal Column Mk. A. Total Dead and Super Load (B/7.) = 309 85 tons

Super Load

Roof $\frac{60^{\#} \times 20' \times 15'}{2240}$ = 8·04

Floors $8/\frac{100^{\#} \times 20' \times 15'}{2240}$ = 107·20 — 115·24 "

Dead Load. = 194·61 "

External Column Mk. B Total Dead and Super Load (B/12) = 258·88 tons

Super Load:

Roof $\frac{60^{\#} \times 20' \times 10'}{2240}$ = 5·36

Floors $8/\frac{100^{\#} \times 20' \times 10'}{2240}$ = 71·46 — 76·82 "

Dead Load = 182·06 "

Stability Moment due to dead load on external windward column about base of external leeward column L (Fig 30)

= 182 06 × 50·0' = 9103·0 ton-ft

So that stability moment provided by dead load on external columns alone is far greater than overturning moment due to wind.

# See Design Sheet C/2

(*a*) the wind forces are resisted entirely by the frames of the building.

(*b*) the beam-to-column connections or joints of the frames are rigid.

(*c*) the point of contraflexure in each storey-length of the columns occurs at mid-height of the storey.

(*d*) the lengths of all members remain unchanged.

(*e*) the joints of the frames at each floor remain level.

(*f*) the horizontal shear in each storey is shared by all the columns in proportion to their stiffnesses in that storey.

In making such assumptions the designer should be aware of the extent of their validity. For example, the walls and floors do have a stiffening effect on the framework that is neglected in the design. Again, the joints assumed to be rigid under (*b*) above are the same pinned connections assumed in the primary design of the beams and columns by the simple method for vertical loading. In fact, the type of connection commonly used, together with the concrete casing to the beams and columns, impart a stiffness or restraint to the joints which, while not making them completely rigid, modify the moment distribution for both vertical and horizontal loading to an indeterminate extent. Assumption (*c*) is frequently modified in the case of the first storey so that the point of contraflexure is considered to be at one-third of the height from the base. While neither assumption is universally true, it can be shown that in this case they introduce no serious error, and the same may be said of (*d*) and (*e*). In view of (*d*) and (*e*) the deflections of all columns at a floor, due to horizontal wind forces, must be equal, and consequently the columns in any storey contribute shear resistance to the wind in proportion to their stiffnesses. A main frame of the building under consideration when deflected by the wind will assume the shape shown, much exaggerated, in Fig. 31.

The wind pressure on the vertical face of the nine-storey building of Design Example (B) may be regarded as a system of horizontal forces acting on the main frames at the roof- and floor-beam levels (C/5); their magnitudes for any one frame being equal to the intensity of the pressure multiplied by the vertical panel areas. On Design Sheet C/2 it is seen that since the external and internal columns are of the same section above the 3rd floor, the horizontal shear is divided equally between the four columns in each storey. In the 3rd, 2nd and 1st storeys the internal columns are stiffer than the external columns and consequently are credited with a larger proportion of the total shear in the storey. In Fig. 32 shear forces in each column are indicated at the assumed points of contraflexure where the moments are zero

(C/5). The bending moments at the top and bottom of the columns in each storey are equal to the column shear multiplied by the distance from the point of contraflexure to the frame joints. For equilibrium at each external joint the bending moment in the beam must be equal to the sum of the moments in the storey lengths of the column above and below the beam. The sum of the moments in the columns at the internal joints are shared between the beams of the main frame meeting at each joint in proportion to their stiffnesses or I/L—values (see Design Sheet C/3).

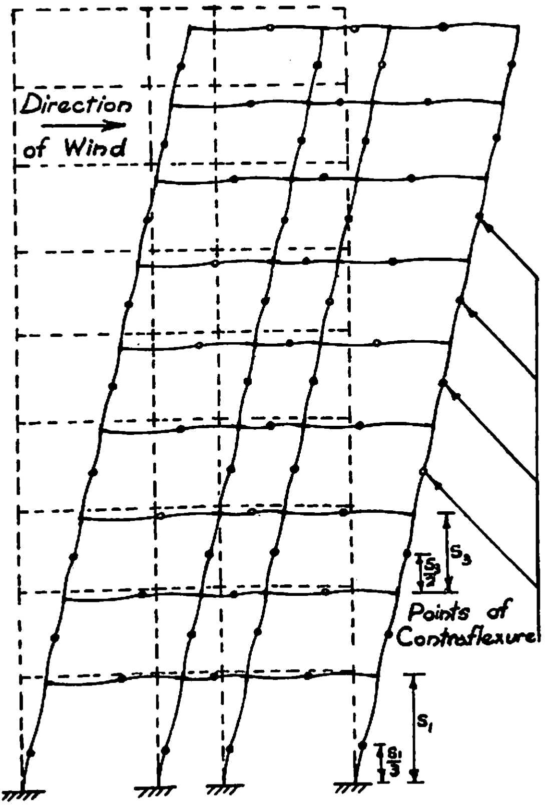


Fig. 31.

C/2

Wind Stress Analysis of a 3-Bay 9-Storey Steel Building Frame

Horizontal wind pressure in accordance with Cl 11(c) of B.S. 449 (1948) is given by.

$$p = \frac{v^2}{600}\frac{\sqrt{1+0\cdot06(h-s)}}{2240} = \frac{62\cdot5^2}{600}\frac{\sqrt{1+0\cdot06\times92}}{2240} = 0\cdot0074 \text{ tons/ft}^2$$

when v = 62·5 m.p.h (wind velocity)

h = 119'-0" (height of parapet above ground)

s = 27'-0" (sheltered height above ground)

Horizontal Wind Forces and Shear Forces on Columns.

	Wind Force (tons)	Horizontal Shear (tons) on External Column.	Internal Column.
Roof	0·0074 × 20·0' × 9·0' = 1·33		
9th Storey		$\frac{1\cdot33}{4}$ = 0·33	0·33
8th Floor	0·0074 × 20·0' × 12·0' = 1·77		
8th Storey		$\frac{3\cdot10}{4}$ = 0·77	0·77
7th Floor	0·0074 × 20·0' × 12·5' = 1·84		
7th Storey		$\frac{4\cdot94}{4}$ = 1·24	1·24.
6th Floor	0·0074 × 20·0' × 13·0' = 1·92		
6th Storey		$\frac{6\cdot86}{4}$ = 1·72	1·72
5th Floor	1·92		
5th Storey		$\frac{8\cdot78}{4}$ = 2·20	2·20
4th Floor	1·92		
4th Storey		$\frac{10\cdot70}{4}$ = 2·68	2·68
3rd Floor	1·92		
3rd Storey		$12\cdot62 \times \frac{0\cdot86}{1\cdot86}$ = 2·92	3·39
2nd Floor	0·0074 × 20·0' × 6·5' = 0·96		
2nd Storey		$13\cdot58 \times \frac{0\cdot86}{1\cdot86}$ = 3·14	3·65
1st Floor	Nil		
1st Storey		$13\cdot58 \times \frac{0\cdot74}{1\cdot74}$ = 2·88	3·91

Relative Stiffness of Columns

(See Design Sheets B/9 to B/13)

	3rd & 2nd Storeys	1st Storey
$\frac{I_{xx} \text{ of External Column}}{I_{xx} \text{ of Internal Column}}$ =	$\frac{1585\cdot81}{1848\cdot82}$ = 0·86	$\frac{1848\cdot82}{2510\cdot46}$ = 0·74

C/3

Relative Stiffness of Beams

Roof Beams :
$$\frac{\text{Stiffness of Outer Span}}{\text{Stiffness of Inner Span}} = \frac{491{\cdot}91}{146{\cdot}23} \times \frac{10}{20} = 1{\cdot}68$$

Floor Beams .
$$\frac{\text{Stiffness of Outer Span}}{\text{Stiffness of Inner Span}} = \frac{725{\cdot}05}{221{\cdot}07} \times \frac{10}{20} = 1{\cdot}64$$

Shear forces at points of contraflexure in columns are accompanied by direct forces in beams and by bending moments in columns at frame joints. Balancing moments are induced in beams at each joint in proportion to the stiffnesses of the beams.

Bending Moments in Columns and Beams (ton-ft. units)

	Columns		Beams		
	External.	Internal	Outer Span at:		Inner Span
			Ext. Col.	Int. Col	
Roof			2·00	$2{\cdot}00 \times \frac{1{\cdot}68}{2{\cdot}68} = 1{\cdot}25$	0·75
9th Storey	0·33 × 6·0* = 2·00	2·00*			
8th Floor			6·62	$6{\cdot}62 \times \frac{1{\cdot}64}{2{\cdot}64} = 4{\cdot}11$	2·51
8th Storey	0·77 × 6·0 = 4·62	4·62			
7th Floor			12·68	$12{\cdot}68 \times \frac{1{\cdot}64}{2{\cdot}64} = 7{\cdot}88$	4·80
7th Storey	1·24 × 6·5* = 8·06	8·06			
6th Floor			19·24	$19{\cdot}24 \times \frac{1{\cdot}64}{2{\cdot}64} = 11{\cdot}96$	7·28
6th Storey	1·72 × 6·5 = 11·18	11·18			
5th Floor			25·48	$25{\cdot}48 \times \frac{1{\cdot}64}{2{\cdot}64} = 15{\cdot}85$	9·63
5th Storey	2·20 × 6·5 = 14·30	14·30			
4th Floor			31·72	$31{\cdot}72 \times \frac{1{\cdot}64}{2{\cdot}64} = 19{\cdot}70$	12·02
4th Storey	2·68 × 6·5 = 17·42	17·42			
3 Floor			36·40	$39{\cdot}45 \times \frac{1{\cdot}64}{2{\cdot}64} = 24{\cdot}50$	14·95
3rd Storey	2·92 × 6·5 = 18·98	3·39 × 6·5 = 22·03			
2nd Floor			39·39	$45{\cdot}75 \times \frac{1{\cdot}64}{2{\cdot}64} = 28{\cdot}40$	17·35
2nd Storey	3·14 × 6·5 = 20·41	3·65 × 6·5 = 23·72			
1st Floor			53·05	$68{\cdot}03 \times \frac{1{\cdot}64}{2{\cdot}64} = 42{\cdot}30$	25·73
1st Storey	2·88 × 11·33* = 32·64	3·91 × 11·33* = 44·31			

* Points of contraflexure assumed at mid-height of columns in 2nd to 9th storeys, and at $\frac{1}{3}$rd-height from base in columns of 1st storey.

C/4.

Direct Stresses in Beams. (tons)

	Outer Windward Bay	Inner Bay	Outer Leeward Bay
Roof	1.33 − 0.33 = 1.00	− 0.33 = 0.77	− 0.33 = 0.44
8th Fl.	1.77 + 0.33 − 0.77 = 1.33	+ 0.33 − 0.77 = 0.89	− 0.33 − 0.77 = 0.45
7th "	1.84 + 0.77 − 1.24 = 1.37	+ 0.77 − 1.24 = 0.90	= 0.43
6th "	1.92 + 1.24 − 1.72 = 1.44	+ 1.24 − 1.72 = 0.96	= 0.48
5th "	1.92 + 1.72 − 2.20 = 1.44	+ 1.72 − 2.20 = 0.96	= 0.48
4th "	1.92 + 2.20 − 2.68 = 1.44	+ 2.20 − 2.68 = 0.96	= 0.48
3rd "	1.92 + 2.68 − 2.92 = 1.68	+ 2.66 − 3.39 = 0.97	= 0.26
2nd "	0.96 + 2.92 − 3.14 = 0.74	+ 3.39 − 3.65 = 0.48	= 0.26
1st "	3.14 − 2.88 = 0.26	+ 3.65 − 3.91 = 0	= − 0.26

Shear Forces on Beams and Direct Forces on Columns (tons.)

	Beams		Columns.	
	Outer Span	Inner Span	External	Internal.
Roof	$\frac{2.00+1.25}{20} = 0.16$	$\frac{0.75\times 2}{10} = 0.15$		
9th Storey			0.16	0.01
8th Floor	$\frac{6.62+4.11}{20} = 0.54$	$\frac{2.51\times 2}{10} = 0.50$		
8th Storey			0.70	0.05
7th Floor	$\frac{12.68+7.88}{20} = 1.03$	$\frac{4.80\times 2}{10} = 0.96$		
7th Storey			1.73	0.12
6th Floor	$\frac{19.24+11.96}{20} = 1.56$	$\frac{7.28\times 2}{10} = 1.46$		
6th Storey			3.29	0.22
5th Floor	$\frac{25.48+16.86}{20} = 2.07$	$\frac{9.63\times 2}{10} = 1.93$		
5th Storey			5.36	0.36
4th Floor	$\frac{31.72+19.70}{20} = 2.57$	$\frac{12.02\times 2}{10} = 2.40$		
4th Storey			7.93	0.53
3rd Floor	$\frac{36.40+24.50}{20} = 3.04$	$\frac{14.95\times 2}{10} = 2.99$		
3rd Storey.			10.97	0.58
2nd Floor	$\frac{39.39+28.40}{20} = 3.39$	$\frac{17.35\times 2}{10} = 3.47$		
2nd Storey			14.36	0.50
1st Floor	$\frac{53.05+42.30}{20} = 4.77$	$\frac{25.73\times 2}{10} = 5.15$		
1st Storey			19.13	0.12

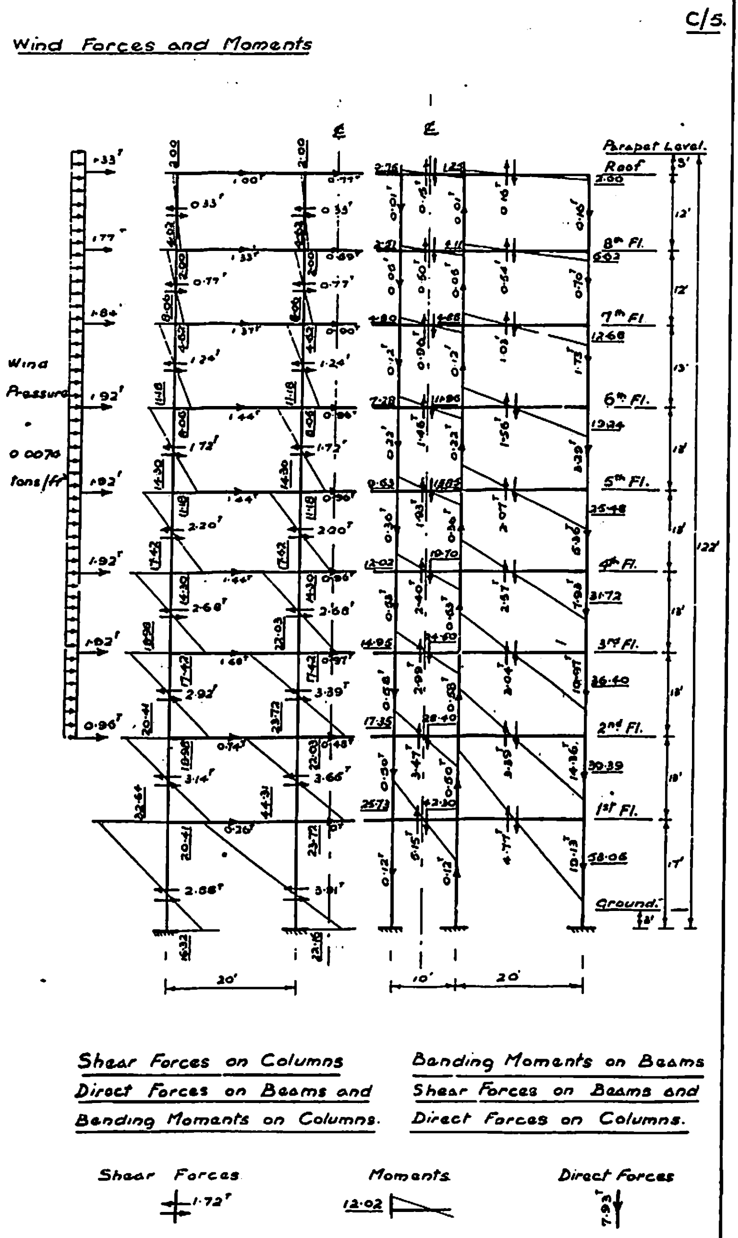

C/5.
Wind Forces and Moments
Wind Pressure
0.0070 tons/ft²
Parapet Level.
Roof
8th Fl.
7th Fl.
6th Fl.
5th Fl.
4th Fl.
3rd Fl.
2nd Fl.
1st Fl.
Ground.
20'
10'
20'
122'
Shear Forces on Columns
Direct Forces on Beams and
Bending Moments on Columns.
Bending Moments on Beams
Shear Forces on Beams and
Direct Forces on Columns.
Shear Forces
Moments
Direct Forces
1·72ᵀ
12·02
7·93ᵀ

Fig. 32.

The columns to the leeward of the external wall subjected to the wind pressure receive their share of the total shear in each storey from the beams which as a result are in compression. Thus, on Design Sheet C/4, the direct compressive force on each beam is equal to the panel load less the shear in the columns at the windward end of the beam.

The shear forces on the beams are calculated from the bending moments at each end, and the direct forces on each storey length of the columns follow from the sum of the beam shears at each joint (C/4). The wind forces on the frame and the induced moments, shear forces and direct forces in the beams and columns are recorded in Fig. 32 on Design Sheet C/5.

Beams and Columns Subjected to Wind Forces.—The capacity of the beams and columns to withstand the wind forces on the frame, in addition to the dead and superimposed loads, is investigated on Design Sheets B/17 and B/18. In the case of the beams the maximum combined bending moment is seen to exceed the simple mid-span moment by less than 25% so that the beam-section selected by the simple method of design for dead and superimposed loads alone is adequate, according to Clause 25 of B.S. 449. The small direct compressive stress in the beams has been neglected.

For the columns the same procedure is followed as in their primary design for dead and superimposed loads. The maximum permissible stresses F_a and F_{bc} for the columns may exceed the values employed for the primary design in equation (ii), viz. :—

$$\frac{f_a}{F_a} + \frac{f_{bc}}{F_{bc}} \not> 1$$

by 25% when the wind moments and stresses are included with those due to dead and superimposed loads. Using these enhanced values for F_a and F_{bc}, the column sections adopted in the primary design are also adequate.

A criticism of the method employed on Design Sheet B/17 might be that the idealised bending moment diagrams for the dead and superimposed loading do not indicate the inevitable restraining moments that the connections must cause at the ends of the beams, with the result that the beams are probably over-designed while the connections are under-designed. This aspect will be considered further in the design of the beam-to-column connections.

Beam-to-Column Connections.—In the absence of other forms of bracing, the beam-to-column connections of tall building frames must be capable of resisting the frame moments induced by wind

B/17

Beams Subjected to Wind Forces in addition to Vertical Loading

Check for Maximum Moment in 1st Floor Beams Mk. F1., F2. etc.

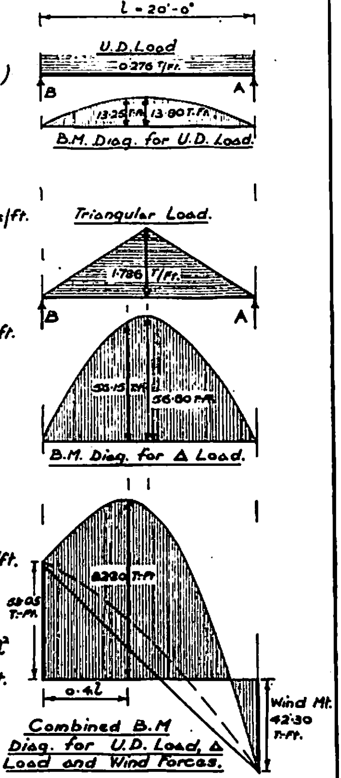


Fig. 33.

(a) Due to U.D Load (Design Sheet B/5)
Mt. at a distance x' from external column B is

$M_x = R_B \cdot x - \frac{w.x^2}{2}$

where $R_B = \frac{w.l}{2}$ $\quad l = 20$

and $w = \frac{12{,}360}{20 \times 2240} = 0.276$ tons/ft.

at	$x = 0.4l$	$0.5l$
	$M_x = 0.120 \times w.l^2$	$0.125 \times w.l^2$
	$= 13.25$ ton-ft	13.80 ton.-ft.

(b) Due to Triangular Panel Load (B/4.)
Mt. at x' from column B is

$M_x = R_B.x - \frac{w_x.x^2}{6}$

where $R_B = \frac{w_{max.}}{4}.l$, $\quad l = 20'$

$w_x = w_{max} \cdot \left(\frac{x}{l/2}\right)$

and $w_{max.} = \frac{200^{\#} \times 20'}{2240} = 1.786$ tons/ft.

at	$x = 0.4l$	$0.5l.$
	$M_x = 0.079 \times w_{max} l^2$	$0.083 \times w_{max} l^2$
	$= 56.15$ ton-ft.	59.50 ton-ft.

(c) Due to Wind Forces (C/3)

$M_B = 53.05$ ton.-ft $\qquad M_A = 42.30$ ton.-ft.

and M_x is zero at $x = \frac{53.05}{53.05 + 42.30} \times 20' = 11.10'$ from B

at	$x = 0.4l$	$0.5l$
	$M_x = \frac{53.05 \times 3.10}{11.10}$	$\frac{53.05 \times 1.10}{11.10}$
	$= 14.80$ ton.-ft.	5.25 ton-ft.

By inspection of the combined B.M. Diag. for (a), (b) and (c) the max. mt. on the beam occurs almost at the 0.4l point from the external column B, and

Total $M_{0.4l} = 13.25 + 56.15 + 14.80 = 82.20$ ton-ft.

Since this max. combined mt does not exceed the normal max. mt. without wind by more than 25% the beam adopted is adequate

B/18

Columns Subjected to Wind Forces in addition to Vertical Loading

Check on Maximum Direct Compression and Bending Stresses.

Main Internal Column. Mk. A.

Above 5th Floor.

Max Direct Load (Design Sheet B/8) 119·56 tons

Direct Wind Force (C/4) = 0·22 · Total = 119·78 tons

$f_a' = \frac{119{\cdot}78}{22{\cdot}58} = 5{\cdot}31$ tons/in² $F_a' = 7{\cdot}35 + 25\% = 9{\cdot}19$ tons/in²

Eccentricity Mt. = 31·50 ton.-ins

Wind Mt. = 11·18×12 = 134·20 · Total. 165·70 ton.-ins

$f_{bc}' = \frac{166{\cdot}70}{123{\cdot}0} = 1{\cdot}35$ tons/in² $F_{bc}' = 10{\cdot}0 + 25\% = 12{\cdot}5$ tons/in².

and $\frac{f_a'}{F_a'} + \frac{f_{bc}'}{F_{bc}'} = \frac{5{\cdot}31}{9{\cdot}19} + \frac{1{\cdot}35}{12{\cdot}5} = 0{\cdot}686$ – okay!

Base to 1st Floor.

Max Direct Load. (Design Sheet B/10) = 266·97 tons

Direct Wind Force (C/4) = 0·12 Total = 267·09 tons

$f_a' = \frac{267{\cdot}09}{52{\cdot}09} = 5{\cdot}14$ tons/in² $F_a' = 6{\cdot}94 + 25\% = 8{\cdot}68$ tons/in²

Eccen. Mt. (1st Fl.) = 42·30 ton-ins

Wind Mt. = 44·31×12 = 531·72 · Total. 574·02 ton-ins.

$f_{bc}' = \frac{574{\cdot}02}{309{\cdot}5} = 1{\cdot}86$ tons/in² $F_{bc}' = 12{\cdot}5$ tons/in².

and $\frac{f_a'}{F_a} + \frac{f_{bc}'}{F_{bc}'} = \frac{5{\cdot}14}{8{\cdot}68} + \frac{1{\cdot}86}{12{\cdot}5} = 0{\cdot}747$ – okay!

Main External Column. Mk. B.

Base to 1st Floor.

Max Direct Load (Design Sheet B/13) = 230·24 tons

Direct Wind Force (C/4) = 19·13 · Total 249·37 tons.

$f_a' = \frac{249{\cdot}37}{41{\cdot}59} = 6{\cdot}00$ tons/in² $F_a' = 6{\cdot}94 + 25\% = 8{\cdot}68$ tons/in²

Eccen. Mt. (1st Fl.) = 37·80 ton-ins

Wind Mt = 32·64×12 = 391·68 · Total 429·48 ton-ins.

$f_{bc}' = \frac{429{\cdot}48}{238{\cdot}70} = 1{\cdot}80$ tons/in² $F_{bc}' = 12{\cdot}5$ tons/in².

and $\frac{f_a'}{F_a'} + \frac{f_{bc}'}{F_{bc}'} = \frac{6{\cdot}00}{8{\cdot}68} + \frac{1{\cdot}80}{12{\cdot}5} = 0{\cdot}835$ – okay!

18

forces. The connections are often designed to resist the frame moments due to the wind forces alone. For moments at the joints in excess of these the connections are expected to yield or give by deformation of the cleats, as indicated in Fig. 34. It will be borne in mind that the deformation must be greatly exaggerated to illustrate it at all in a sketch.

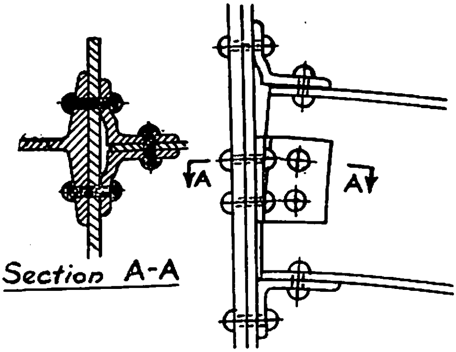


Fig. 34.

The main beam-to-column connections in Design Example (B) (Design Sheets B/19-B/22) consist of web-cleats with sufficient rivets to transfer the end reactions to the columns, and flange connections which are proportioned to withstand the moments imparted to the columns due to eccentricity of the end connections in addition to the frame moments induced by the action of the wind forces. According to the approximate wind-stress analysis (Design Sheets C/2-C/5) the beams of the main frame are subjected to end moments that increase from the roof to 1st floor, so that it would appear at first sight that different end connections should be provided for the beams at each floor. However, it is convenient to reduce the number of different connections employed by grouping as indicated on Design Sheet B/22.

Present day specifications permit an increase in the maximum design stresses when wind forces are considered in addition to normal loading ; the increase being 33% above the normal maximum in the case of the A.I.S.C. Specification(1) and 25% in B.S. 449 (1948). Advantage is taken of this increase in stresses in the design of the main beam-to-column connections in Example (B) (Design Sheets B/19-B/21). .

Certain field rivets will be stressed in tension in the types of moment-connections employed. The question of rivets subject to direct tension is extremely complex, and it has been common in

B/19

Beam-to-Column Connections

8th Floor Beams Mk. F1., F2. etc

(Adopt same connections for Roof Beams Mk R1. R2 etc.)

Max shear due to vertical loading (Design Sheet B/5) = 11·50 tons
" " " " wind (C/4) = 0·54 "
Total vertical force at connection = 12·04 "

Since shear force due to wind is less than 25% of shear at end connection due to vertical loading, web cleats and rivets adopted to transfer normal max. shear will be sufficient for combined effects of normal max vertical loading and wind.

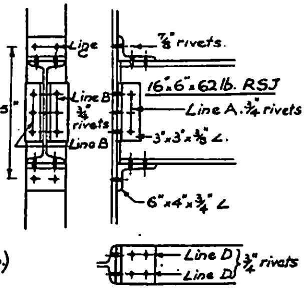


Fig. 35.

Adopt 3"×3"×$\frac{3}{8}$" web-cleats.

N° of $\frac{3}{4}$" shop rivets requ'd. on Line A

in bearing $= \frac{11{\cdot}50}{0{\cdot}812 \times 0{\cdot}55 \times 12{\cdot}0} = 3$ (min.)

N° of $\frac{3}{4}$" field rivets requ'd. on Lines B :

in bearing $= \frac{11{\cdot}50}{0{\cdot}812 \times 0{\cdot}38 \times 10{\cdot}0} = 4$ (min.)

in s. shear $= \frac{11{\cdot}50}{\pi/4 \times 0{\cdot}812^2 \times 5{\cdot}0} = 5$ (min.)

Max. Mt. due to eccentricity (B/5) = 11·50 × 8·0" = 92·00 ton-ins
" " " " wind (C/3) = 6·62 × 12 = 79·44 "
Total Mt. at connection. = 171·44 "

Assume flange ∠-cleats with 4" legs upstanding for moment connections, (see Fig. 35.)

Tension in rivets connecting upstanding legs of cleats to column

$= \frac{171{\cdot}44}{20{\cdot}5} = 8{\cdot}37$ tons.

At a max. tensile stress in field rivets of 5·0 + 25% = 6·25 tons/in²

N° of $\frac{7}{8}$" field rivets requ'd. on Line C $= \frac{8{\cdot}37}{\pi/4 \times 0{\cdot}937^2 \times 6{\cdot}25} = 2$ (min.)

At a max. shear stress in field rivets of 5·0 + 25% = 6·25 tons/in²

N° of $\frac{7}{8}$" field rivets requ'd. on Lines D $= \frac{171{\cdot}44}{16{\cdot}0(\frac{\pi}{4} \times 0{\cdot}937^2 \times 6{\cdot}25)} = 4$ (min.)

Thickness of Flange Cleats.

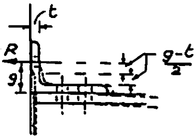


Fig. 36.

Assume $t = 0{\cdot}75$", then if s is the stress due to bending

$$R.\left(\frac{g-t}{2}\right) = \left(\frac{b.t^2}{6}\right).s$$

where $R = 8{\cdot}37$ tons
$g = 2{\cdot}25$"
width $b = 6{\cdot}0$" (gross)

then $s = \frac{8{\cdot}47.(2{\cdot}25 - 0{\cdot}75).6{\cdot}0}{2 \times 6{\cdot}0 \times 0{\cdot}75^2} = 11{\cdot}30$ tons/in²

and max. permissible value of s = 10·0 + 25% = 12·50 tons/in²

B/20

Beam - to - Column Connections. (Cont'd.)

3rd Floor Beams. Mk. F1., F2. etc.

(Adopt same connections for corresponding beams of 4th & 5th Fls.)

Max shear due to vertical loading (B/5) = 11·69 tons.

" " " wind (C/4) = 3·04 "

Total vertical force at connection = 14·73 "

Web connections as for 8th Fl. beams Mk. F1., F2. etc. are sufficient for total vertical force.

Max. Mt. due to eccentricity

(B/9) = 11·69 × 8·75" = 102·30 ton-ins.

Max. Mt. due to wind

(C/3) = 36·40 × 12 = 436·80 "

Total Mt. at connection = 539·10 "

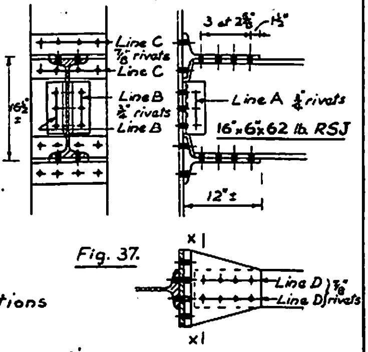


Fig. 37.

Assume split-beam flange connections out of a 24" × 7½" × 95 lb. RSJ.

Mt.-arm at connection = 16" + 0·57" say 16·5"

Flange force F at connection $= \frac{539 \cdot 10}{16 \cdot 5}$ = 32·65 tons

No of 7/8" field rivets requ'd in tension on Lines C

$$= \frac{32 \cdot 65}{\pi/4 \times 0 \cdot 937^2 \times 6 \cdot 25} = 8 \text{ (min.)}$$

No of 7/8" field rivets requ'd. in s. shear on Lines D

$$= \frac{539 \cdot 10}{16 \cdot 0 (\pi/4 \times 0 \cdot 937^2 \times 6 \cdot 25)} = 8 \text{ (min.)}$$

Thickness of Flange of Split-beam Connections.

For the split beam adopted av. thickness of flange t_f = 1·011"

then if f_b is the stress in flange due to bending, (Fig. 38)

$$\frac{F}{2}\left(\frac{c}{4}\right) = \left(\frac{b \cdot t_f^2}{6}\right) \cdot f_b$$

where F = 32·65 tons

c = 4·50"

width b = 12" (gross)

then $f_b = \frac{32 \cdot 65 (4 \cdot 5) \cdot 6}{2 \times 4 \times 12 \times 1 \cdot 011^2}$ = 8·98 tons/in².

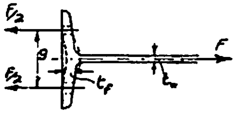


Fig. 38.

Web of Split-beam Connections.

At a max. tensile stress on web of split beam of 9·00 + 25% = 11·25 tons/in².:

Net width requ'd on section X-X (Fig. 37).

$$= \frac{32 \cdot 65}{0 \cdot 57 \times 11 \cdot 25} = 5 \cdot 10''$$

The 12" width of spit-beam web can be reduced over a length of 12" to accommodate 4 rivets on Lines D.

M

B/21

Beam-to-Column Connections (Cont'd.)

1st Floor Beams Mk. F1., F2. etc

(Adopt same connections for corresponding beams of 2nd Fl.)

Max shear due to vertical loading (B/5) — 11·69 tons

" " " " wind. (C/4) — 4·77 "

Total vertical force at connection " — 16·46 "

Web connections as for 8th Fl. beams Mk F1., F2 etc are sufficient for total vertical force

Max. Mt. due to eccentricity (B/13) — $11{\cdot}69 \times 8{\cdot}75''$ — 102·30 ton-ins.

Max. Mt " " wind (C/3) — $53{\cdot}06 \times 12$ — 636·60 "

Total Mt. at connection — 738·90 "

With the same split-beam flange connections as for corresponding beams of 3rd Fl. the flange force would be:

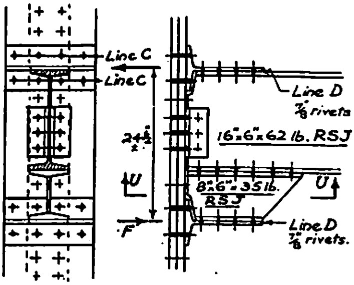
$$F = \frac{738{\cdot}90}{16{\cdot}5} = 44{\cdot}80 \text{ tons.}$$

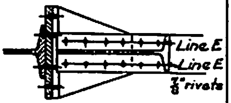
which would require

$$\frac{44{\cdot}80}{\frac{\pi}{4} \times 0{\cdot}937^2 \times 6{\cdot}25} = 10{\cdot}4$$

i.e. a minimum of 12 No 7/8" rivets (for symmetry) in tension on Lines C. Mt-arm at connection must be increased so that reduced flange force may be transmitted to column by 8 No 7/8" rivets on lines C at a tensile stress not exceeding 6·25 tons/in². Employing an 8"x 6" x 35 lb. RSJ stool in connections (see Fig. 39), the flange force

$$F = \frac{738{\cdot}90}{24{\cdot}5} = 30{\cdot}10 \text{ tons}$$





Section UU

Fig. 39.

No of 7/8" field rivets in tension on Lines C

$$= \frac{30{\cdot}10}{\frac{\pi}{4} \times 0{\cdot}937^2 \times 6{\cdot}25} = 8 \text{ (min.)}$$

No. of 7/8" field rivets in s. shear on Lines D

$$= \frac{738{\cdot}90}{24{\cdot}0\left(\frac{\pi}{4} \times 0{\cdot}937^2 \times 6{\cdot}25\right)} = 8 \text{ (min.)}$$

No of 7/8" field rivets in s. shear on Lines E

$$= \frac{738{\cdot}90}{16\left(\frac{\pi}{4} \times 0{\cdot}937^2 \times 6{\cdot}25\right)} = 11 \text{ (min.)}$$

Web of RSJ Stool.

For an 8"x 6" x 35 lb. RSJ t_w — 0·35"

At a max shear stress of 6·5 + 25 % — 8·12 tons/in².

on web of stool, the min'm. length required (approximately)

$$= \frac{30{\cdot}10}{0{\cdot}35 \times 8{\cdot}12} = 10{\cdot}6''$$

, adopt a length of 12" to accommodate 8 No 7/8" rivets on Lines D, increasing to 17·5" to accommodate 12 No 7/8" rivets on Lines E.

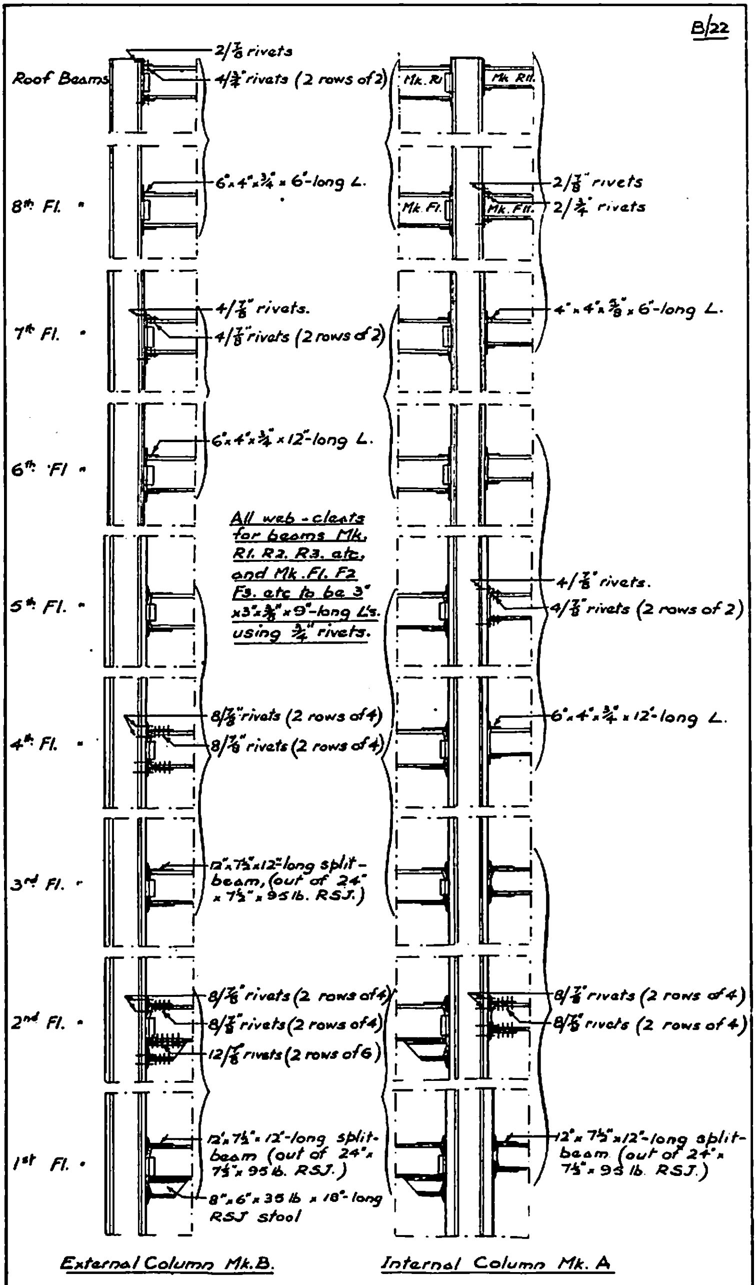

B/22
2/⅞ rivets
Roof Beams
4/⅞" rivets (2 rows of 2)
Mk. R1
Mk. R11.
6"×4"×¾"×6"-long L.
8th Fl. "
2/⅞" rivets
Mk. F1.
Mk. F11.
2/¾" rivets
4/⅞" rivets.
7th Fl. "
4/⅞" rivets (2 rows of 2)
4"×4"×⅝"×6"-long L.
6"×4"×¾"×12"-long L.
6th Fl "
All web-cleats for beams Mk. R1. R2. R3. etc. and Mk. F1. F2 F3. etc to be 3" ×3"×⅜"×9"-long L's. using ¾" rivets.
5th Fl. "
4/⅞" rivets.
4/⅞" rivets (2 rows of 2)
8/⅞" rivets (2 rows of 4)
4th Fl. "
8/⅞" rivets (2 rows of 4)
6"×4"×¾"×12"-long L.
12"×7½"×12"-long split-beam, (out of 24" × 7½" × 95 lb. RSJ.)
3rd Fl. "
8/⅞" rivets (2 rows of 4)
2nd Fl. "
8/⅞" rivets (2 rows of 4)
12/⅞" rivets (2 rows of 6)
8/⅞" rivets (2 rows of 4)
8/⅞" rivets (2 rows of 4)
1st Fl. "
12"×7½"×12"-long split-beam (out of 24"×7½"×95 lb. RSJ.)
8"×6"×35 lb × 18"-long RSJ stool
12"×7½"×12"-long split-beam (out of 24"×7½"×95 lb. RSJ.)
External Column Mk. B.
Internal Column Mk. A

Fig. 40. Beam-to-Column Connections.

the past for Specifications either to prohibit the use of rivets in tension, or to omit reference to permissible tensile stresses in rivets. One reason for this caution is, perhaps, the knowledge that a considerable initial tension must exist in hot-driven rivets irrespective of any external force.

In the Specifications of the A.I.S.C. the permissible stress for rivets in tension has been increased from 13,500 lb. per sq. in., in the Second Edition of 1934, to 20,000 lb. per sq. in., in the Fifth Edition of 1949, which suggests an increased confidence in the ability of rivets to resist tension. The maximum stress for field rivets in tension permitted in the 1948 Edition of B.S. 449 is 4 tons per sq. in. This British Standard is under review, however, at the present time and it is considered likely that revisions will be made in the permissible stresses. In Design Sheets B/19-B/21 a basic stress of 5 tons per sq. in. has been adopted for field rivets in tension.

Field connections may also be made by means of turned and fitted bolts. These permit the use of higher design stresses in both tension and shear, and although more expensive, are in many respects superior to field driven rivets.

The simple theory of flexure is applied to determine the thickness of the angle-cleats and split-beam connections in bending as indicated in Figs. 36 and 38.

The methods of design for the beam-to-column connections of Design Examples (A) and (B) have been employed for many years, and few failures in building frames have been attributed to inadequate connections. Even so it is important that the designer should appreciate the empirical nature of the design and that the actual distribution of stress in such connections is quite indeterminate. Tests have indicated that split-beam connections of the type employed at the 3rd floor of Design Example (B) are much less flexible than the angle-cleats at the 5th floor and above, and it is probable that, with sufficient care in design, connections incorporating split-beam flange clips may be regarded as fully rigid. Professor T. C. Shedd(5) treats the subject of beam-to-column connections at some length, and reference will be made with benefit to his book on steel design.

Column Base.

A gusseted base is designed to transmit the direct load and bending moment from the internal column Mk. A to its foundation in Design Sheets B/23-B/24 which, in general, are self-explanatory. Rivets are provided to transmit 60% of the direct load and the full bending moment from the column shaft to the gussets and from the gussets to the cleats in accordance with Clause 50 (*c*) (i) of B.S. 449.

When several plates and sections are riveted together it is necessary to check the grip lengths of the connecting rivets as, for example, on Line AA, Fig. 43. The grip length normally should not exceed four diameters (see B.S. 449, Clause 42 (*f*)).

Since bearing pressure occurs over the full area of the base plate holding-down bolts are not required. However, some means of locating the column-base and holding it in position are required and are provided by bolts set in the concrete foundation and grouted in after the column-base is set in position.

PART III.

FULLY RIGID DESIGN.

Limitations of Simple Design.—In Part I, under "Methods of Design," three alternative methods were referred to, and the first of these—the simple method has been employed in Design Examples (A) and (B).

In the design of a column by the simple method it is first necessary to consider the restraint at the ends in order to arrive at a value for the effective length l of the column. This value is then combined with the least radius of gyration of the section proposed for the column in order to determine the maximum permissible compressive stress under axial load. The stresses in the column due to small bending moments caused by the eccentric reactions from the beams are also calculated, and the column is regarded as satisfactory if the sum of the two ratios of actual to maximum permissible stresses for direct compression and bending does not exceed unity (see Eq. (ii)).

Assessing the effective length of a column is an uncertain process since it is no more possible, with the types of connection commonly employed to determine the extent to which the beams restrain the ends of the columns in each storey of a multi-storey building, than it is to foretell the true restraining moment that will be present at the ends of the beams. Again, the value of l is fixed in simple design quite irrespective of any bending moment imposed on the columns at the frame joints. In cases where such moments give rise to bending stresses in the columns that are very small in comparison with the direct stress this may not be serious. However, when the true bending moments on the columns are appreciable, as must often be the case with the semi-rigid type of connection, the accompanying rotation of the ends of the column will result in it having a curved shape quite different from that assumed implicitly in ascribing a value to l.

B/23

Base of Internal Column Mk A

Max direct load on column (Design Sheet B/10) V_1 = 266·97 tons
Min " " " (C/1) V_2 = 194·61 "
Max shear force on column (C/2) H = 3·91 "
Mt at 1st Fl level due to eccentricity of loading is regarded as ineffective at base of column
Max wind mt at column base (C/5) M = 22·16 ton-ft
Direct force on internal columns due to wind is negligible
At a max bearing pressure of 40 tons/ft² on concrete foundation.
Min. area of base plate $= \frac{266·97}{40} = 6·68$ ft²

With say ½"-gussets and angle-cleats with 4" horizontal leg.
Length of base plate d = 16·25" + 2/0·5" + 2/4" = 25·25" (min.) = say 2'-6"
and width $b = \frac{6·68}{2·50}$ = 2·67' (min) = say 3'-3"

For max. wind. mt. with full super. load:
Max. bearing pressure $= \frac{266·97}{3·25 \times 2·50} + \frac{6 \times 22·16}{3·25 \times 2·50^2} = 32·90 + 6·54 = 39·54$ tons/ft
Min. bearing pressure = $= 32·90 - 6·54 = 26·36$ "
For max. wind mt. without super. load:
Max. bearing pressure $= \frac{194·61}{3·25 \times 2·50} + \frac{6 \times 22·16}{3·25 \times 2·50^2} = 24·00 + 6·54 = 30·54$ "
Min. bearing pressure = $= 24·00 - 6·54 = 17·46$ "

so that bearing pressure pressure occurs over entire area of base plate and fixed-end condition for columns is realised under all combinations of loading.
Gussets and cleats to be attached square to and flush with end of column and whole assembly to be machined for bearing on plate.
Rivets to be provided to transfer 60% of direct load and full bending mt. to base-plate

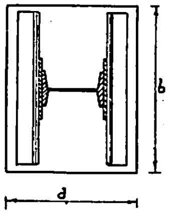


Fig. 41

Force at each flange of column due to wind mt.
$= \frac{22·16 \times 12}{16·25} = 16·38$ tons
N° of ⅞" shop rivets in s. shear to transmit wind-mt at column flanges $= \frac{16·38}{\frac{\pi}{4} \times 0·938^2 \times (6·0 + 26\%)} = 4$ (min.)

N° of ⅞" shop rivets in s shear to transmit 60% of direct load to column base $= \frac{266·97 \times 0·60}{\frac{\pi}{4} \times 0·938^2 \times 6·0} = 40$ (min)

Employ ⅞" rivets in column base as follows:
6 N° to connect cleats to web of column (d shear)
18 N° to connect each gusset to flange of column (s shear)
and 18 N° to connect angle-cleat to gusset
Rivets connecting cleats to base plate to be c'sunk on underside

HH

B/24

Base of Internal Column Mk A (Cont'd.)

Thickness of Base Plate

Plate regarded as a simple beam on two supports A and B at centres of gussets. and subjected to loading corresponding to bearing pressure due to max direct load V_1 and bending moment M on base

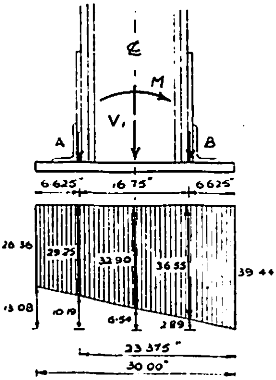


Fig. 42.

Intensity of pressure at A

$$= 26.36 + (39.44 - 26.36) \times \frac{6.625}{30.0}$$

$$= 29.36 \text{ tons/ft}^2$$

About A :

$$R_B \times 16.75 = \frac{29.25 \times 23.375^2}{144 \times 2} + \frac{10.19 \times 23.375^2}{144 \times 3}$$

or $R_B = \dfrac{23.375^2}{16.75 \times 144}\left(\dfrac{29.25}{2} + \dfrac{10.19}{3}\right)$ = 4.07 tons per inch width

Per inch width of plate :

Mt. at ℄ $= \dfrac{32.90 \times 15.0^2}{144 \times 2} + \dfrac{6.54 \times 15.0^2}{144 \times 3} - 4.07 \times 8.375$ = 4.99 Ton-ins

Mt. at B $= \dfrac{36.55 \times 6.625^2}{144 \times 2} + \dfrac{2.89 \times 6.625}{144 \times 3}$ = 5.85 "

Min. thickness of plate t_b at a bending stress of 12.0 tons/in² is given by = $\dfrac{bt_b^2}{6} = \dfrac{5.85}{12.0}$ or $t_b = \sqrt{\dfrac{5.85}{2}}$ = 1.71"

Adopt a plate $1\frac{3}{4}$" thick.

Thickness of Gussets – must be sufficient to ensure that value of a $\frac{9}{8}$" rivets on it is not less than s. shear value. i.e.

$$t_g \times 0.938'' \times 12.0 \nless \frac{\pi}{4} \times 0.938^2 \times 6.0$$

or $t_g \nless 0.37''$ – Adopt $\frac{1}{2}$"-thick gussets.

Thickness of Angle-cleats. Since the upward force at windward flange of column is counter-acted by direct column load there will be no bending of cleats. Vertical leg of 6" is requ'd. to accommodate $\frac{7}{8}$" rivets on two gauge lines Horizontal leg of 4" is preferred to accomodate 1" bolts Adopt 6" × 4" × $\frac{1}{2}$" ∠'s

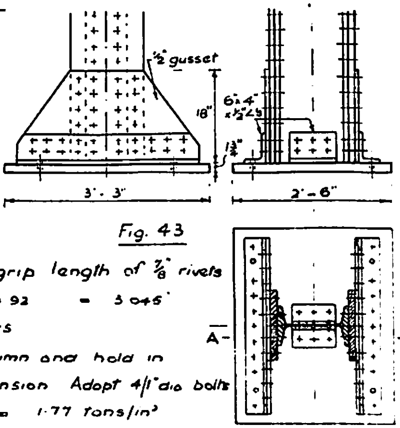


Fig. 43

Grip Length of Rivets – max. grip length of $\frac{7}{8}$" rivets on Line AA = 0.50 + 0.50 + 1.125 + 0.92 = 3.045"

i.e. less than 4 diameters

Bolts. – requ'd to locate column and hold in position Not subjected to tension Adopt 4/1" dia bolts

Shear stress $= \dfrac{3.91}{4(\frac{\pi}{4} \times 0.840^2)}$ = 1.77 tons/in²

It has been seen that for the main frames of a multi-storey building to resist wind forces, where diagonal bracing is ruled out, the beam-to-column connections must withstand the induced frame moments. Now these are the same connections, the effects of which were ignored in the design of the beams and columns under vertical loading. Thus, it might appear that connections which have to be able to offer resistance to wind forces, must deform and be quite incapable of resisting the bending of the beams under vertical loads. The fact is that an end moment is almost always operative on the beams but is ignored in simple design. If it were only that, as a consequence, the beams are over-designed, the criticism would be on the grounds of economy there only. However, it follows that the restraining moments which the semi-rigid connections produce at the ends of the beams must be withstood by the columns, so that for vertical loading, too, a bending moment is applied to the columns although it is ignored in simple design. Moreover, it is possible that the end-connections may be under-designed.

B.S. 449 in Clause 29 (*a*) states that the simple method of design "applies to structures in which the end-connections between members are such that they will not develop restraint moments adversely affecting the members and the structure as a whole", so that all connections of beams are virtually pin-jointed. The inference here is that if a tall building must depend on the rigidity of its frame joints in order to resist non-vertical forces such as wind pressure, then for such a building design by the simple method is inappropriate. From this it should not be thought that the simple method of design for tall buildings is necessarily dangerous, and there are, of course, in existence many multi-storey buildings that have been designed by this method. The question rather is whether a more rational method of design should not be sought which will provide the designer with a surer knowledge of the stress-distribution throughout the framework of the building while, at the same time, affording savings in steel. With these objectives in mind recourse might be had to either the semi-rigid or fully-rigid methods of design referred to in the British Standard. It is stipulated that semi-rigid design shall be in conformity with the "Recommendations for Design" in the Final Report of the Steel Structures Research Committee, 1936. Much useful information is contained in the report from test on steel frames in addition to a description of a semi-rigid design procedure that represents a considerable advance over simple design.

Rigid Design.—From the design examples and discussion so far it has been seen that the entire system of columns and beams of a tall steel building is, in fact, a frame that will resist loading of any nature as a whole, with an interaction between the vertical and horizontal members which takes place through the connections or joints. The riveted connections commonly used in steel building frames are semi-rigid, that is they will deform or "slip" a certain amount when the frame is subject to load so that the rotation of the end of each member at a joint is not necessarily proportional to the moment induced there by the loading. It is possible virtually to eliminate this slip at the joints by careful detailing of the riveted connections or by welding. The result is a rigid frame that exhibits a high degree of indeterminacy and for which the design process is quite different from that followed in the simple design method.

Methods of indeterminate analysis have gained ground in the last thirty years and one of these—the method of slope deflection—has been used and expounded successfully for the design of tall frame buildings by Messrs. J. I. Parcel and G. A. Maney,(6) to whose writings the student may well refer. In Example (D) a slope-deflection method devised by J. E. Goldberg (7) is used to redesign the nine-storey, three-bay building frame of Design Example (B).

Referring to Fig. 44, which shows four members of a rigid framework meeting at a common joint, it is clear that when one member, say AB, is subjected to load the ends of the member will tend to rotate and will be opposed in this by other members meeting at the joints. Since the joint A is rigid the ends of all members meeting there will rotate through the same angle to reach the equilibrium position for the loading considered, and bending moments will be set up in the ends of each member at the joint in proportion to their stiffnesses. A certain proportion of the bending moment in each member will be transferred to the joints at the far end of the members from A so that they, too, will tend to rotate due to the loading applied to AB. By the method of slope-deflection the rotations and hence the moments in all members of a frame can be

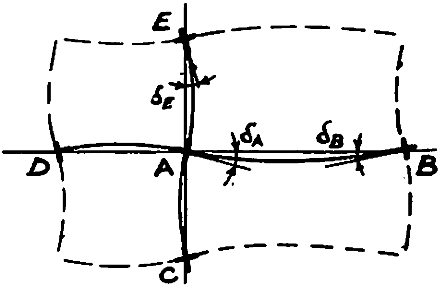


Fig. 44.

calculated for any condition of loading providing the stiffnesses of the members are known.

The general slope deflection relationship for any member AB, viz. :—

$$M_{AB} = M_{F.AB} - \frac{2.E.I}{L}(2\delta_A + \delta_B) \quad \dots \quad \dots \quad \dots \text{(iii)}$$

may be re-written as

$$M_{AB} = M_{F.AB} - K_{AB}.(2.\theta_A + \theta_B) \quad \dots \quad \dots \quad \dots \quad \dots \text{(iv)}$$

where M_{AB} is the true moment at A in member AB of a rigid frame.

$M_{F.AB}$ is the fixed end moment at A, *e.g.*, $\frac{w.L^2}{12}$, $\frac{P.L.}{8}$, etc., depending on the loading. For a member that is not subjected to transverse loading, the fixed end moment is, of course, zero.

I and L are the moment of inertia and length of the member, and E is the modulus of elasticity.

δ_A and δ_B are the angular rotations at the ends A and B in radians, and θ_A and θ_B are the end rotations multiplied by the quantity

$$\left[\frac{2.E.I}{L}\right].\frac{1}{K_{AB}}$$

In Equation (iv) K_{AB} is a measure of the stiffness of AB, being the ratio of the quantity $\frac{2E.I}{L}$ for the member AB to a corresponding quantity for a standard member of the frame.

Then at the joint A in Fig. 44 we have :—

$$\begin{aligned}
M_{AB} &= M_{F.AB} - K_{AB}\,.\,(2\theta_A + \theta_B)\\
M_{AC} &= M_{F.AC} - K_{AC}\,.\,(2\theta_A + \theta_C)\\
M_{AD} &= M_{F.AD} - K_{AD}\,.\,(2\theta_A + \theta_D)\\
M_{AE} &= M_{F.AE} - K_{AE}\,.\,(2\theta_A + \theta_E)
\end{aligned}$$

Adding : $\Sigma M = \Sigma M_F - \Sigma K.(2\theta_A) - \Sigma(K\theta_X)$

But the algebraic sum of the moments ΣM at any joint must be zero for equilibrium, so that

$$\Sigma M_F = \Sigma K(2\theta_A) + \Sigma(K\theta_X) \quad \dots \quad \dots \quad \dots \quad \dots \quad \text{(v)}$$

for each joint of the frame under the system of loading considered. This equation may be solved for θ_A by rewriting it as

$$\theta_A = \frac{\Sigma M_F - \Sigma(K\theta_X)}{2(\Sigma K)} \quad \dots \quad \dots \quad \text{(vi)}$$

in which the only unknowns are the θ_x's. It is evident from Equation (vi) that the value of θ for any joint is in the neighbourhood of $\frac{\Sigma M_F}{2(\Sigma K)}$ for that joint, since it is in fact $\frac{\Sigma M_F}{2(\Sigma K)}$ plus or minus a small correction due to the rotation of the adjacent joints.

With the three-bay, nine-storey frame which is to be analysed in mind, the procedure is as follows :—

(*a*) The K-values of all members are determined. The beam and column sections have been assumed in Design Example (D) (Design Sheet D/1) from a knowledge of the sections required in simple design. These K-values are entered in Table 1 on Design Sheet D/2.

(*b*) The 2.(ΣK)-values for all joints of the frame are computed. These appear in the circles of the frame joints in Fig. 45.

(*c*) The fixed end moments M_F for the dead load and superimposed loading are computed. In Design Example (D) the full superimposed loading on all roof- and floor-beams is used, and the moments written in position above the beams in Fig. 45, regard being paid to the sign convention adopted, viz. : (+) for clockwise moments and rotations ; (−) for counter-clockwise moments and rotations. In this example the moments at the external joints of the frame due to the eccentricity of the wall beams about the axis of the columns perpendicular to the plane of the frame are also taken into account.

(*d*) The frame joints are separated into two alternate groups. For a symmetrical frame loaded symmetrically about the centre line, it is sufficient to consider one half of the frame only. In Fig. 45 the joints C, F, G, K, L, O, P, S, T form Group I, and Group II is composed of joints D, E, H, J, M, N, Q, R, U.

(*e*) The values of $\theta_I = \frac{\Sigma M_F - \Sigma(K\theta_x)}{2\Sigma K}$ are computed for all joints of group I, assuming that $\theta_x = \frac{\Sigma (M_F)}{2\Sigma(K)}$.

(*f*) The values of $\theta_{II} = \frac{\Sigma M_F - \Sigma(K\theta_{x(I)})}{2\Sigma K}$ are next computed for all joints of group II (see Design Sheet D/3).

(*g*) The computations of (*e*) and (*f*) are repeated using the values of θ_x obtained in successive operations. The values of θ for each joint are seen to converge ; the more times the process is repeated the greater being the accuracy obtained. In Example (D) the computations were made three times for each group of joints (D/3 and D/4).

(*h*) The end moments are determined for beams and columns by substituting the $\theta-$ values in equations corresponding to Equation (iv) for each member. These substitutions appear on Design Sheet D/5 for the beams and on D/6 for the columns. Since the columns are not subjected to transverse loading in the case considered M_F for each column length is zero. The final distribution of moments throughout the frame is indicated in Fig. 46, Design Sheet D/6, and at this stage a useful check on the computations is available in that the sum of the moments at any joint should approach zero.

It remains for the stresses due to the moments obtained from this analysis to be checked to ensure that the trial members are sufficient, and in this check a knowledge of the frame moments due to wind forces is also required. To determine the wind moments a further slope deflection analysis of the frame may be made by the "Maney-Goldberg" method(6) with a suitable correction for side sway. In the check on beam and column stresses on Design Sheets D/7 and D/8 values of the wind moments as determined in Design Example (C) are taken into account.

Beams.—In the case of the beams a normal maximum bending stress of 10.0 tons/in. is increased by 25% when wind stresses are present. It is evident that in a tall building frame the wind moments on the beams are of paramount importance, and that the reductions in the size of the main beams in the upper storeys as compared with simple design are not possible in the lower storeys.

Columns.—The stress distribution in a column subject to end bending moments in addition to axial load has been the subject of much experimentation and theoretical research. In a paper by Prof. J. F. Baker and P. D. Holder(8) the investigations concern a strut which is assumed to have an initial curvature approximating to a circular arc and which is subjected to end moments not necessarily of the same magnitude or sign. Equations are developed relating the total maximum permissible stress in the strut to the ratio of end bending stresses by means of which it is possible to estimate the maximum bending stress that the strut can withstand for a given direct axial stress.

For design purposes the results obtained from these equations are presented in the form of curves in a further paper(9). Both double and single curvature of the columns are considered, and to provide for either eventuality composite curves have been prepared

D/1

Design of a 3-Bay 9-Storey Rigid Steel Building Frame

(Dimensions and loading as for frame of Design Example B)

Member	Minimum Section Required in Simple Design	I_{xx} (in^4)	Trial Section in Rigid Design.	I_{xx} (in^4)
Internal Column				
1st Storey	14"×8"×70 lb RSJ	705·58	12"×8"×65 lb RSJ	487·77
	2/14"×1⅛" Pls.	1804·88	2/12×1⅛" Pls	1165·68
	Total I_{xx} =	2510·36		= 1653·45
4th Storey	14"×6"×46 lb RSJ	442·57	12"×6"×44 lb RSJ	316·76
	2/12"×¾" Pls	979·92	2/10"×¾" Pls	610·30
	Total I_{xx} =	1422·49		= 927·06
8th Storey	14"×6"×46 lb. RSJ	442·57	12"×6"×44 lb RSJ	316·76
External Column.				
1st Storey	14"×8"×70 lb RSJ	705·58	12"×8"×65 lb. RSJ	487·77
	2/14"×¾" Pls.	1143·24	2/12"×¾" Pls.	732·36
	Total I_{xx} =	1848·82		= 1220·13
4th Storey	14"×6"×46 lb RSJ	442·57	12"×6"×44 lb. RSJ	316·76
	2/12"×¾" Pls.	979·92	2/10"×¾" Pls	610·30
	Total I_{xx} =	1422·49		= 927·06
8th Storey	14"×6"×46 lb RSJ	442·57	12"×6"×44 lb. RSJ.	316·76
Roof Beams Mk. R11	10"×5"×30 lb. RSJ	146·23	10"×5"×30 lb. RSJ.	146·23
Mk. R1	15"×5"×42 lb RSJ	428·49	13"×5"×35 lb RSJ	283·51
Floor Beams Mk. F11,	12"×5"×32 lb RSJ	221·07	12"×5"×32 lb RSJ	221·07
Mk. F1.				
8th, 7th, 6th, & 5th Fls.	16"×6"×62 lb RSJ	725·05	15"×6"×45 lb RSJ	491·91
4th & 3rd Fls	16"×6"×62 lb RSJ	725·05	16"×6"×50 lb RSJ	618·09
2nd & 1st Fls	16"×6"×62 lb RSJ	725·05	16"×6"×62 lb RSJ	725·05

In Table 1 the K-values denote the ratios of the quantity $\frac{2.E.I}{L}$ for members of the frame to the same quantity for the Roof Beam Mk R11. (here designated UU')

H

D/2

Table 1. –
Stiffness Ratios.

$E = 13{,}000$ tons/in²

Member	I. in⁴	L in.	$\frac{2.E.I}{L}$	K.
UU′	146·23	120	31,650	1·0
SS′	221·07	120	47,900	1·5
QQ′	221·07	120	47,900	1·5
OO′	221·07	120	47,900	1·5
MM′	221·07	120	47,900	1·5
KK′	221·07	120	47,900	1·5
HH′	221·07	120	47,900	1·5
FF′	221·07	120	47,900	1·5
DD′	221·07	120	47,900	1·5
TU	283·51	240	30,750	1·0
RS	491·91	240	53,300	1·7
PQ	491·91	240	53,300	1·7
NO	491·91	240	53,300	1·7
LM	491·91	240	53,300	1·7
JK	618·09	240	67,000	2·1
GH	618·09	240	67,000	2·1
EF	725·05	240	78,600	2·5
CD	725·05	240	78,600	2·5
SU	316·76	144	57,200	1·6
QS	316·76	144	57,200	1·8
OQ	603·96	156	100,650	3·2
MO	603·96	156	100,650	3·2
KM	927·06	156	154,500	4·9
HK	927·06	156	154,500	4·9
FH	1,220·13	156	203,500	6·4
DF	1,220·13	156	203,500	6·4
BD	1,653·45	204	210,500	6·7
RT	316·76	144	57,200	1·8
PR	316·76	144	57,200	1·8
NP	603·96	156	100,650	3·2
LN	603·96	156	100,650	3·2
JL	927·06	156	154,500	4·9
GJ	927·06	156	154,500	4·9
EG	1,049·12	156	174,900	5·5
CE	1,049·12	156	174,900	5·5
AC	1,220·13	204	155,700	4·9

Moments at external joints due to eccentricity of wall-beam reactions on columns. (B/3 – B/6)

At T: $10{\cdot}44 \times 1{\cdot}5 = 15{\cdot}7$ ton-ins.

At R and P: $17{\cdot}16 \times 1{\cdot}5 = 25{\cdot}8$ "

At N, L, etc. $17{\cdot}80 \times 1{\cdot}5 = 26{\cdot}7$ "

Fixed End Mts. (see B/1 – B/5)

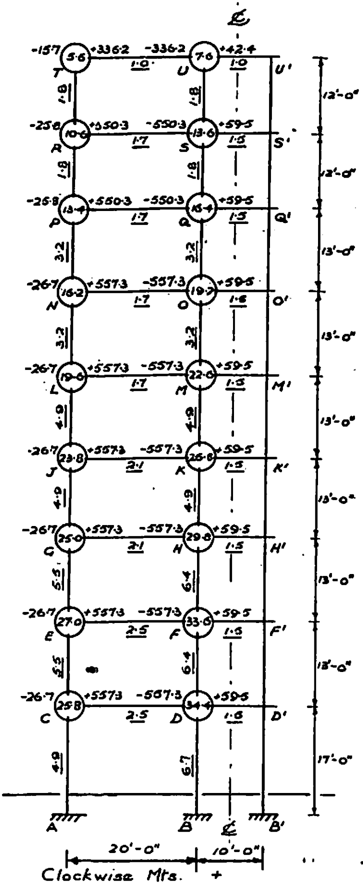
UU′: $\frac{5 \times 1000 \times 120}{48 \times 2240} + \frac{740 \times 120}{12 \times 2240} = 42{\cdot}4$ "

TU: $\frac{5 \times 28000 \times 240}{48 \times 2240} + \frac{2650 \times 240}{12 \times 2240} = 336{\cdot}2$ "

SS′: $\frac{5 \times 10000 \times 120}{48 \times 2240} + \frac{825 \times 120}{12 \times 2240} = 59{\cdot}5$ "

RS: $\frac{5 \times 40000 \times 240}{48 \times 2240} + \frac{11550 \times 240}{12 \times 2240} = 550{\cdot}3$ "

NO: $\frac{5 \times 40000 \times 240}{48 \times 2240} + \frac{12350 \times 240}{12 \times 2240} = 557{\cdot}3$ "



Clockwise Mts. +
Counter- " " –

Fig. 45.

D/3.

Angular Rotation of Joints

1st Approximation

Radians. ×31,650

$$\theta_C = \frac{530{\cdot}6 - \frac{2{\cdot}5}{34{\cdot}4}(-497{\cdot}8) - \frac{5{\cdot}5}{27{\cdot}0}(530{\cdot}6)}{25{\cdot}8} \qquad 17{\cdot}8$$

$$\theta_F = \frac{-497{\cdot}8 - \frac{2{\cdot}5}{27{\cdot}0}(530{\cdot}6) - \frac{6{\cdot}4}{34{\cdot}4}(-497{\cdot}8) - \frac{6{\cdot}4}{29{\cdot}8}(-497{\cdot}8) - 1{\cdot}5(-\theta_{F'})}{33{\cdot}6} = 10{\cdot}8$$

$$\theta_G = \frac{530{\cdot}6 - \frac{2{\cdot}1}{29{\cdot}8}(-497{\cdot}8) - \frac{5{\cdot}5}{27{\cdot}0}(530{\cdot}6) - \frac{4{\cdot}9}{23{\cdot}8}(530{\cdot}6)}{25{\cdot}0} = 13{\cdot}9$$

$$\theta_K = \frac{-497{\cdot}8 - \frac{2{\cdot}1}{25{\cdot}8}(530{\cdot}6) - \frac{4{\cdot}9}{25{\cdot}8}(-497{\cdot}8) - \frac{4{\cdot}9}{22{\cdot}6}(-497{\cdot}8) - 1{\cdot}5(-\theta_{K'})}{26{\cdot}8} = 14{\cdot}0$$

$$\theta_L = \frac{530{\cdot}6 - \frac{1{\cdot}7}{22{\cdot}6}(-497{\cdot}8) - \frac{4{\cdot}9}{23{\cdot}8}(530{\cdot}6) - \frac{3{\cdot}2}{16{\cdot}2}(530{\cdot}6)}{19{\cdot}6} = 18{\cdot}1$$

$$\theta_O = \frac{-497{\cdot}8 - \frac{1{\cdot}7}{16{\cdot}2}(530{\cdot}6) - \frac{3{\cdot}2}{22{\cdot}6}(-497{\cdot}8) - \frac{3{\cdot}2}{16{\cdot}4}(-490{\cdot}8) - 1{\cdot}5(-\theta_{O'})}{19{\cdot}2} = 21{\cdot}9$$

$$\theta_P = \frac{524{\cdot}5 - \frac{1{\cdot}7}{16{\cdot}4}(-490{\cdot}8) - \frac{3{\cdot}2}{16{\cdot}2}(530{\cdot}6) - \frac{1{\cdot}8}{10{\cdot}6}(524{\cdot}5)}{13{\cdot}4} = 28{\cdot}5$$

$$\theta_S = \frac{-490{\cdot}8 - \frac{1{\cdot}7}{10{\cdot}6}(524{\cdot}5) - \frac{1{\cdot}8}{16{\cdot}4}(-490{\cdot}8) - \frac{1{\cdot}8}{7{\cdot}6}(-293{\cdot}8) - 1{\cdot}5(-\theta_{S'})}{13{\cdot}6} = 37{\cdot}4$$

$$\theta_T = \frac{320{\cdot}5 - \frac{1{\cdot}0}{7{\cdot}6}(-293{\cdot}8) - \frac{1{\cdot}8}{10{\cdot}6}(-524{\cdot}5)}{5{\cdot}6} = 48{\cdot}4$$

$$\theta_D = \frac{-497{\cdot}8 - 2{\cdot}5(17{\cdot}8) - 6{\cdot}4(-10{\cdot}8) - 1{\cdot}5(-\theta_{D'})}{34{\cdot}4} = 14{\cdot}4$$

$$\theta_E = \frac{530{\cdot}6 - 2{\cdot}5(-10{\cdot}8) - 5{\cdot}5(17{\cdot}8) - 5{\cdot}5(13{\cdot}9)}{27{\cdot}0} = 14{\cdot}2$$

$$\theta_H = \frac{-497{\cdot}8 - 2{\cdot}1(13{\cdot}9) - 6{\cdot}4(-10{\cdot}8) - 4{\cdot}9(-14{\cdot}0) - 1{\cdot}5(-\theta_{H'})}{29{\cdot}8} = 13{\cdot}7$$

$$\theta_J = \frac{530{\cdot}6 - 2{\cdot}1(-14{\cdot}0) - 4{\cdot}9(13{\cdot}9) - 4{\cdot}9(18{\cdot}1)}{23{\cdot}8} = 17{\cdot}0$$

$$\theta_M = \frac{-497{\cdot}8 - 1{\cdot}7(18{\cdot}1) - 4{\cdot}9(-14{\cdot}0) - 3{\cdot}2(-21{\cdot}9) - 1{\cdot}5(-\theta_{M'})}{22{\cdot}6} = 18{\cdot}5$$

$$\theta_N = \frac{530{\cdot}6 - 1{\cdot}7(-21{\cdot}9) - 3{\cdot}2(18{\cdot}1) - 3{\cdot}2(28{\cdot}5)}{16{\cdot}2} = 25{\cdot}8$$

$$\theta_Q = \frac{-490{\cdot}8 - 1{\cdot}7(28{\cdot}5) - 3{\cdot}2(-21{\cdot}9) - 1{\cdot}8(-37{\cdot}4) - 1{\cdot}5(-\theta_{Q'})}{16{\cdot}4} = 26{\cdot}9$$

$$\theta_R = \frac{524{\cdot}5 - 1{\cdot}7(-37{\cdot}4) - 1{\cdot}8(28{\cdot}5) - 1{\cdot}8(48{\cdot}4)}{10{\cdot}6} = 42{\cdot}5$$

$$\theta_U = \frac{-293{\cdot}8 - 1{\cdot}0(48{\cdot}4) - 1{\cdot}8(-37{\cdot}4) - 1{\cdot}0(-\theta_{U'})}{7{\cdot}6} = 41{\cdot}5$$

D/4.

Angular Rotation of Joints (Cont'd.)

	2nd. Approximation		3rd Approx'n.
		Radians × 31,650.	
θ_C	$=\dfrac{530{\cdot}6-2{\cdot}5(-14{\cdot}4)-5{\cdot}5(14{\cdot}2)}{25{\cdot}8}$	$=18{\cdot}9$	$19{\cdot}1$
θ_F	$=\dfrac{-497{\cdot}8-2{\cdot}5(14{\cdot}2)-6{\cdot}4(-14{\cdot}4)-6{\cdot}4(-13{\cdot}7)-1{\cdot}5(-\theta_F')}{33{\cdot}6}$	$=11{\cdot}0$	$11{\cdot}0$
θ_G	$=\dfrac{530{\cdot}6-2{\cdot}1(-13{\cdot}7)-5{\cdot}5(14{\cdot}2)-4{\cdot}9(17{\cdot}0)}{25{\cdot}0}$	$=15{\cdot}9$	$16{\cdot}4$
θ_K	$=\dfrac{-497{\cdot}8-2{\cdot}1(17{\cdot}0)-4{\cdot}9(-13{\cdot}7)-4{\cdot}9(-18{\cdot}5)-1{\cdot}5(-\theta_K')}{26{\cdot}8}$	$=14{\cdot}8$	$14{\cdot}9$
θ_L	$=\dfrac{530{\cdot}6-1{\cdot}7(-18{\cdot}5)-4{\cdot}9(17{\cdot}0)-3{\cdot}2(25{\cdot}8)}{19{\cdot}6}$	$=20{\cdot}2$	$20{\cdot}5$
θ_O	$=\dfrac{-497{\cdot}8-1{\cdot}7(25{\cdot}8)-3{\cdot}2(-18{\cdot}5)-3{\cdot}2(-26{\cdot}9)-1{\cdot}5(-\theta_O')}{19{\cdot}2}$	$=22{\cdot}4$	$22{\cdot}3$
θ_P	$=\dfrac{524{\cdot}5-1{\cdot}7(-26{\cdot}9)-3{\cdot}2(25{\cdot}8)-1{\cdot}8(42{\cdot}5)}{13{\cdot}4}$	$=30{\cdot}7$	$31{\cdot}1$
θ_S	$=\dfrac{-490{\cdot}8-1{\cdot}7(42{\cdot}5)-1{\cdot}8(-26{\cdot}9)-1{\cdot}8(-41{\cdot}7)-1{\cdot}5(-\theta_S')}{13{\cdot}6}$	$=36{\cdot}4$	$36{\cdot}1$
θ_T	$=\dfrac{320{\cdot}5-1{\cdot}0(-41{\cdot}7)-1{\cdot}8(42{\cdot}5)}{5{\cdot}6}$	$=51{\cdot}0$	$51{\cdot}5$
θ_D	$=\dfrac{-497{\cdot}8-2{\cdot}5(18{\cdot}9)-6{\cdot}4(-11{\cdot}0)-1{\cdot}5(-\theta_D')}{34{\cdot}4}$	$=14{\cdot}5$	$14{\cdot}5$
θ_E	$=\dfrac{530{\cdot}6-2{\cdot}5(-11{\cdot}0)-5{\cdot}5(18{\cdot}9)-5{\cdot}5(15{\cdot}9)}{27{\cdot}0}$	$=13{\cdot}6$	$13{\cdot}4$
θ_H	$=\dfrac{-497{\cdot}8-2{\cdot}1(15{\cdot}9)-6{\cdot}4(-11{\cdot}0)-4{\cdot}9(-14{\cdot}8)-1{\cdot}5(-\theta_H')}{29{\cdot}8}$	$=13{\cdot}7$	$13{\cdot}8$
θ_J	$=\dfrac{530{\cdot}6-2{\cdot}1(-14{\cdot}8)-4{\cdot}9(15{\cdot}9)-4{\cdot}9(20{\cdot}2)}{23{\cdot}8}$	$=16{\cdot}2$	$16{\cdot}0$
θ_M	$=\dfrac{-497{\cdot}8-1{\cdot}7(20{\cdot}2)-4{\cdot}9(-14{\cdot}8)-3{\cdot}2(-22{\cdot}4)-1{\cdot}5(-\theta_M')}{22{\cdot}6}$	$=18{\cdot}4$	$18{\cdot}5$
θ_N	$=\dfrac{530{\cdot}6-1{\cdot}7(-22{\cdot}5)-3{\cdot}2(20{\cdot}2)-3{\cdot}2(30{\cdot}7)}{16{\cdot}2}$	$=25{\cdot}1$	$25{\cdot}0$
θ_Q	$=\dfrac{-490{\cdot}8-1{\cdot}7(30{\cdot}7)-3{\cdot}2(-22{\cdot}4)-1{\cdot}8(-36{\cdot}4)-1{\cdot}5(-\theta_Q')}{16{\cdot}4}$	$=27{\cdot}2$	$27{\cdot}4$
θ_R	$=\dfrac{524{\cdot}5-1{\cdot}7(-36{\cdot}4)-1{\cdot}8(30{\cdot}7)-1{\cdot}8(51{\cdot}0)}{10{\cdot}6}$	$=41{\cdot}5$	$41{\cdot}2$
θ_U	$=\dfrac{-293{\cdot}8-1{\cdot}0(51{\cdot}0)-1{\cdot}8(-36{\cdot}4)-1{\cdot}0(-\theta_U')}{7{\cdot}6}$	$=42{\cdot}3$	$42{\cdot}5$

Ht

D/5

Simple Mid-Span Moments in Beams (From Design Sheets B/1 to B/5)

TU. $44{\cdot}56 \times 12 = 534{\cdot}7$ ton-ins UU'.: $5{\cdot}62 \times 12 = 67{\cdot}4$ ton-ins

RS. $72{\cdot}38 \times 12 = 868{\cdot}6$ " SS'.: $7{\cdot}89 \times 12 = 94{\cdot}7$ "

NO, etc.: $73{\cdot}29 \times 12 = 879{\cdot}5$ " OO', etc. $7{\cdot}89 \times 12 = 94{\cdot}7$ "

Frame Moments (ton-ins units)

End Moments in Beams

$M_{TU} = 336{\cdot}2 - 1{\cdot}0\left[2(51{\cdot}5) + (-42{\cdot}5)\right] = 275{\cdot}7$

$M_{UT} = -336{\cdot}2 - 1{\cdot}0\left[2(-42{\cdot}5) + (51{\cdot}5)\right] = -302{\cdot}7$

$M_{UU'} = 42{\cdot}4 - 1{\cdot}0\left[2(-42{\cdot}5) + (42{\cdot}5)\right] = 84{\cdot}9$

$M_{RS} = 550{\cdot}3 - 1{\cdot}7\left[2(41{\cdot}2) + (-36{\cdot}1)\right] = 471{\cdot}7$

$M_{SR} = -550{\cdot}3 - 1{\cdot}7\left[2(-36{\cdot}1) + (41{\cdot}2)\right] = -497{\cdot}6$

$M_{SS'} = 59{\cdot}5 - 1{\cdot}5\left[2(-36{\cdot}1) + (36{\cdot}1)\right] = 113{\cdot}6$

$M_{PQ} = 557{\cdot}3 - 1{\cdot}7\left[2(31{\cdot}1) + (-27{\cdot}4)\right] = 491{\cdot}1$

$M_{QP} = -557{\cdot}3 - 1{\cdot}7\left[2(-27{\cdot}4) + (31{\cdot}1)\right] = -510{\cdot}0$

$M_{QQ'} = 59{\cdot}5 - 1{\cdot}5\left[2(-27{\cdot}4) + (27{\cdot}4)\right] = 100{\cdot}6$

$M_{NO} = 557{\cdot}3 - 1{\cdot}7\left[2(25{\cdot}0) + (-22{\cdot}3)\right] = 510{\cdot}2$

$M_{ON} = -557{\cdot}3 - 1{\cdot}7\left[2(-22{\cdot}3) + (25{\cdot}0)\right] = -524{\cdot}0$

$M_{OO'} = 59{\cdot}5 - 1{\cdot}5\left[2(-22{\cdot}3) + (22{\cdot}3)\right] = 92{\cdot}9$

$M_{LM} = 557{\cdot}3 - 1{\cdot}7\left[2(20{\cdot}5) + (-18{\cdot}5)\right] = 519{\cdot}0$

$M_{ML} = -557{\cdot}3 - 1{\cdot}7\left[2(-18{\cdot}5) + (20{\cdot}5)\right] = -529{\cdot}3$

$M_{MM'} = 59{\cdot}5 - 1{\cdot}5\left[2(-18{\cdot}5) + (18{\cdot}5)\right] = 87{\cdot}2$

$M_{JK} = 557{\cdot}3 - 2{\cdot}1\left[2(16{\cdot}0) + (-14{\cdot}9)\right] = 521{\cdot}4$

$M_{KJ} = -557{\cdot}3 - 2{\cdot}1\left[2(-14{\cdot}9) + (16{\cdot}0)\right] = -528{\cdot}3$

$M_{KK'} = 59{\cdot}5 - 1{\cdot}5\left[2(-14{\cdot}9) + (14{\cdot}9)\right] = 81{\cdot}8$

$M_{GH} = 557{\cdot}3 - 2{\cdot}1\left[2(16{\cdot}4) + (-13{\cdot}8)\right] = 517{\cdot}4$

$M_{HG} = -557{\cdot}3 - 2{\cdot}1\left[2(-13{\cdot}8) + (16{\cdot}4)\right] = -533{\cdot}8$

$M_{HH'} = 59{\cdot}5 - 1{\cdot}5\left[2(-13{\cdot}8) + (13{\cdot}8)\right] = 80{\cdot}2$

$M_{EF} = 557{\cdot}3 - 2{\cdot}5\left[2(13{\cdot}4) + (-11{\cdot}0)\right] = 517{\cdot}8$

$M_{FE} = -557{\cdot}3 - 2{\cdot}5\left[2(-11{\cdot}0) + (13{\cdot}4)\right] = -536{\cdot}8$

$M_{FF'} = 59{\cdot}5 - 1{\cdot}5\left[2(-11{\cdot}0) + (11{\cdot}0)\right] = 76{\cdot}0$

$M_{CD} = 557{\cdot}3 - 2{\cdot}5\left[2(19{\cdot}1) + (-14{\cdot}5)\right] = 498{\cdot}1$

$M_{DC} = -557{\cdot}3 - 2{\cdot}5\left[2(-14{\cdot}5) + (19{\cdot}1)\right] = -532{\cdot}6$

$M_{DD'} = 59{\cdot}5 - 1{\cdot}5\left[2(-14{\cdot}5) + (14{\cdot}5)\right] = 81{\cdot}3$

Mid-Span Moments in Beams

TU $534{\cdot}7 - \left(\frac{275{\cdot}7 + 302{\cdot}7}{2}\right) = 245{\cdot}5$

UU' $67{\cdot}4 - 84{\cdot}9 = -17{\cdot}5$

RS $868{\cdot}6 - \left(\frac{471{\cdot}7 + 497{\cdot}6}{2}\right) = 384{\cdot}0$

SS' $94{\cdot}7 - 113{\cdot}6 = -18{\cdot}9$

PQ $868{\cdot}6 - \left(\frac{491{\cdot}1 + 510{\cdot}0}{2}\right) = 368{\cdot}1$

QQ' $94{\cdot}7 - 100{\cdot}6 = -5{\cdot}9$

NO $879{\cdot}5 - \left(\frac{510{\cdot}2 + 524{\cdot}0}{2}\right) = 362{\cdot}4$

OO' $94{\cdot}7 - 92{\cdot}9 = 1{\cdot}8$

LM $879{\cdot}5 - \left(\frac{519{\cdot}0 + 529{\cdot}3}{2}\right) = 355{\cdot}3$

MM' $94{\cdot}7 - 87{\cdot}2 = 7{\cdot}5$

JK $879{\cdot}5 - \left(\frac{521{\cdot}4 + 528{\cdot}3}{2}\right) = 354{\cdot}7$

KK' $94{\cdot}7 - 81{\cdot}8 = 12{\cdot}9$

GH $879{\cdot}5 - \left(\frac{517{\cdot}4 + 533{\cdot}8}{2}\right) = 353{\cdot}9$

HH' $94{\cdot}7 - 80{\cdot}2 = 14{\cdot}5$

EF $879{\cdot}5 - \left(\frac{517{\cdot}8 + 536{\cdot}8}{2}\right) = 352{\cdot}7$

FF' $94{\cdot}7 - 76{\cdot}0 = 18{\cdot}7$

CD $879{\cdot}5 - \left(\frac{498{\cdot}1 + 532{\cdot}6}{2}\right) = 364{\cdot}1$

DD' $94{\cdot}7 - 81{\cdot}3 = 13{\cdot}4$

D/6

End Moments in Columns

$M_{TR} = 0 - 1\cdot8\left[2(51\cdot5) + (41\cdot2)\right] = -259\cdot6$

$M_{RT} = 0 - 1\cdot8\left[2(41\cdot2) + (51\cdot5)\right] = -241\cdot0$

$M_{US} = 0 - 1\cdot8\left[2(-42\cdot5) + (-36\cdot1)\right] = 218\cdot0$

$M_{SU} = 0 - 1\cdot8\left[2(-36\cdot1) + (-42\cdot5)\right] = 206\cdot5$

$M_{RP} = 0 - 1\cdot8\left[2(41\cdot2) + (31\cdot1)\right] = -204\cdot3$

$M_{PR} = 0 - 1\cdot8\left[2(31\cdot1) + (41\cdot2)\right] = -186\cdot1$

$M_{SQ} = 0 - 1\cdot8\left[2(-36\cdot1) + (-27\cdot4)\right] = 179\cdot3$

$M_{QS} = 0 - 1\cdot8\left[2(-27\cdot4) + (-36\cdot1)\right] = 163\cdot6$

$M_{PN} = 0 - 3\cdot2\left[2(31\cdot1) + (25\cdot0)\right] = -279\cdot0$

$M_{NP} = 0 - 3\cdot2\left[2(25\cdot0) + (31\cdot1)\right] = -259\cdot5$

$M_{QO} = 0 - 3\cdot2\left[2(-27\cdot4) + (-22\cdot3)\right] = 246\cdot7$

$M_{OQ} = 0 - 3\cdot2\left[2(-22\cdot3) + (-27\cdot4)\right] = 230\cdot4$

$M_{NL} = 00 - 3\cdot2\left[2(25\cdot0) + (20\cdot5)\right] = -225\cdot6$

$M_{LN} = 0 - 3\cdot2\left[2(20\cdot5) + (25\cdot0)\right] = -211\cdot2$

$M_{OM} = 0 - 3\cdot2\left[2(-22\cdot3) + (-18\cdot5)\right] = 201\cdot9$

$M_{MO} = 0 - 3\cdot2\left[2(-18\cdot5) + (-22\cdot3)\right] = 189\cdot8$

$M_{LJ} = 0 - 4\cdot9\left[2(20\cdot5) + (16\cdot0)\right] = -279\cdot3$

$M_{JL} = 0 - 4\cdot9\left[2(16\cdot0) + (20\cdot5)\right] = -257\cdot3$

$M_{MK} = 0 - 4\cdot9\left[2(-18\cdot5) + (-14\cdot9)\right] = 254\cdot3$

$M_{KM} = 0 - 4\cdot9\left[2(-14\cdot9) + (-18\cdot5)\right] = 236\cdot7$

$M_{JG} = 0 - 4\cdot9\left[2(16\cdot0) + (16\cdot4)\right] = -237\cdot2$

$M_{GJ} = 0 - 4\cdot9\left[2(16\cdot4) + (16\cdot0)\right] = -239\cdot1$

$M_{KH} = 0 - 4\cdot9\left[2(-14\cdot9) + (-13\cdot8)\right] = 213\cdot4$

$M_{HK} = 0 - 4\cdot9\left[2(-13\cdot8) + (-14\cdot9)\right] = 208\cdot3$

$M_{GE} = 0 - 5\cdot5\left[2(16\cdot4) + (13\cdot4)\right] = -254\cdot1$

$M_{EG} = 0 - 5\cdot5\left[2(13\cdot4) + (16\cdot4)\right] = -237\cdot6$

$M_{HF} = 0 - 6\cdot4\left[2(-13\cdot8) + (-11\cdot0)\right] = 247\cdot0$

$M_{FH} = 0 - 6\cdot4\left[2(-11\cdot0) + (-13\cdot8)\right] = 229\cdot1$

$M_{EC} = 0 - 5\cdot5\left[2(13\cdot4) + (19\cdot1)\right] = -252\cdot4$

$M_{CE} = 0 - 5\cdot5\left[2(19\cdot1) + (13\cdot4)\right] = -283\cdot8$

$M_{FD} = 0 - 6\cdot4\left[2(-11\cdot0) + (-14\cdot5)\right] = 233\cdot6$

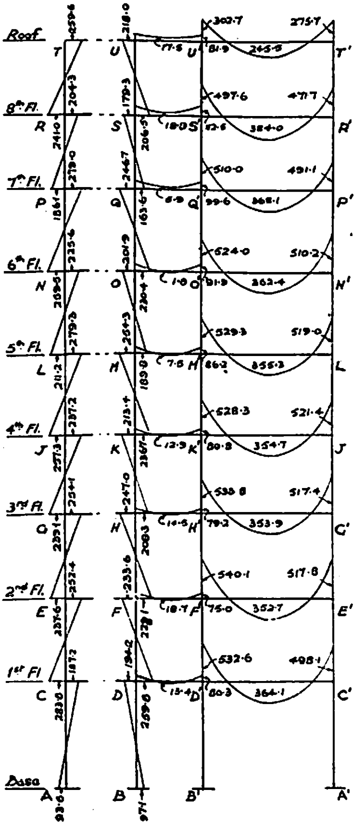
$M_{DF} = 0 - 6\cdot4\left[2(-14\cdot5) + (-11\cdot0)\right] = 256\cdot0$

$M_{CA} = 0 - 4\cdot9\left[2(19\cdot1) + (0)\right] = -187\cdot2$

$M_{AC} = 0 - 4\cdot9\left[2(0) + (19\cdot1)\right] = -93\cdot6$

$M_{DB} = 0 - 6\cdot7\left[2(-14\cdot5) + (-0)\right] = 194\cdot2$

$M_{BD} = 0 - 6\cdot7\left[2(-0) + (-14\cdot5)\right] = 97\cdot1$



Frame Moments due to Vertical Loading.
Fig. 46.

and are reproduced here (in Fig. 47) without considering further their theoretical derivation. It will be noted that the total maximum permissible stress in these curves is limited to 9 tons/in.[2] for small values of L/r, and while this is accepted in checking the

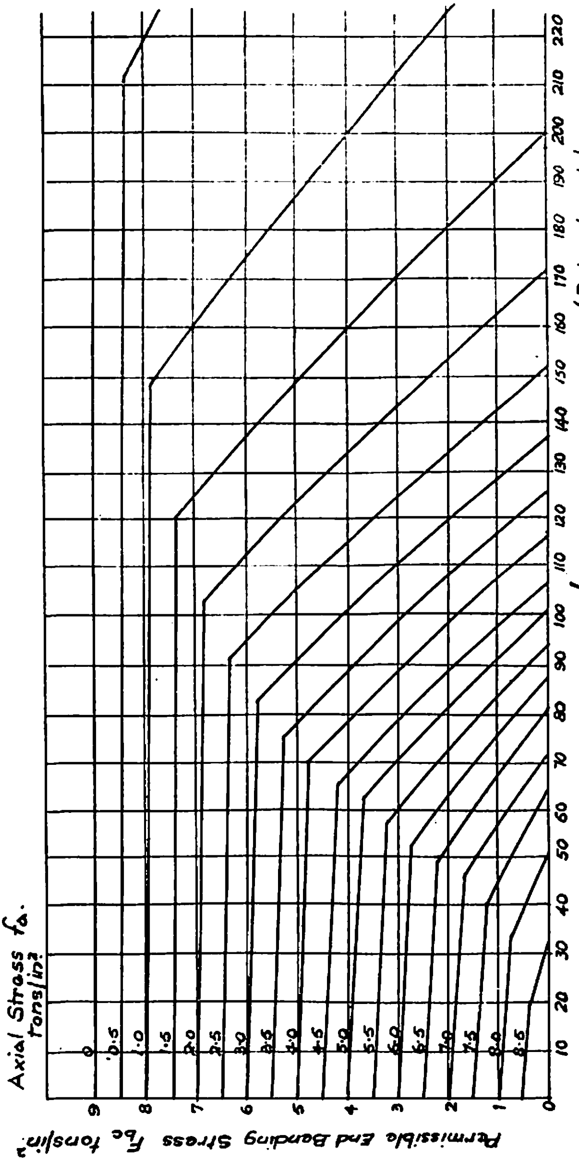


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Fig. 47.- Design Chart for Columns.

stresses in the columns on Design Sheet D/8 it will be appreciated that this relates to plain steel columns, and that with concrete casing an increase to, say, 1·0 tons/in.^{2} would be in order.

In Design Sheet D/8 the "cut-off" value of the length of the column L, obtained by subtracting the depth of the beams from the storey height, is used. Here again the approximate wind moments from Design Example (C) are taken into account and the permissible bending stresses in the columns as obtained from the design chart (Fig. 47) increased by 25%. Since the computed values of the bending stress (f'_{bc}) are less than the permissible values (F'_{bc}), the trial column sections are adequate.

Welded Connections.—In Design Sheet D/9 welded frame joints at the 8th and 3rd floors are designed for the moments due to vertical loading and wind forces acting together. The web of the column at the joints is strengthened by means of horizontal plates welded between the flanges as shown in Figs. 48 and 49.

Economy in Steel.

Beams.—The chief saving in steel by the rigid method of design in Design Example (D) is in the main beams of the 5th to 8th floors. In the outer bays of the frame at these floors a 15″ × 6″ × 45 lb./ft. R.S.J. is sufficient when the frame moments are taken into consideration compared with the simple method in Design Example (B), according to which a 16″ × 6″ × 62 lb./ft. R.S.J. is required.

Columns.—The basic section for the columns in the rigid frame analysis is a 12″ R.S.J. compared with a 14″ R.S.J. in the simple design. However, a revision of Design Example (B) indicates that a basic 12″ R.S.J. section would be adequate for the columns there as well, so that the only saving effected in the columns in Design Example (D) is in the width of the flange plates. These in Design Example (B) are 14″ wide in the storeys having a 14″ × 8″ R.S.J. section and 12″ wide in the storeys having a 14″ × 6″ R.S.J., whereas in Design Example (D) the corresponding widths of the column flanges are 12″ and 10″. An important consideration in selecting the wider flange plates in Design Example (B) is that the moment connections to the columns of the lower floor beams require four rivets in line (see Figs. 37 and 39). In the design of the welded connections of Design Example (D) the length of fillet welds required to resist the moments at the joints can be accommodated within the 12″ or 10″ width of the column flange plates.

Following a redesign of the columns of the three-bay, nine-storey building of Design Example (B) by the simple method with a basic 12″ R.S.J. the estimated total weight of beams and columns of a main frame is 17·4 tons. For the frame of Design Example (D)

D/7.

Check on Stresses in Beams

Roof Beam Mk. R11. (UU', Fig. 46)

Z for trial section = 29·25 in³

Max. Mt. due to dead and superimposed loads (D/6) = 81·9 ton-ins.

$f_b = \frac{81.9}{29.25}$ = 2·8 tons/in²

Roof Beam Mk. R1. (TU, Fig. 46)

Z for trial section = 43·62 in³.

Max. Mt. due to dead and superimposed loads (D/6) = 302·7 ton-ins.

$f_b = \frac{302.7}{43.62}$ = 6·95 tons/in².

1st to 8th Floor-Beams Mk. F11. (SS' etc.)

Z for trial section = 36·84 in³

Max. Mt. due to dead and superimposed loads (D/6) = 112·5 ton-ins.

$f_b = \frac{112.5}{36.84}$ = 3·06 tons/in².

Max. Wind Mt. on 1st Fl. beam (approx., C/5) 25·73×12 = 308·76 ton-ins.

Corresponding Mt. due to dead and super. loads (D/6) = 80·3 "

Max. combined Mt. on 1st Fl. beam. = 389·06 "

$f_b' = \frac{389.06}{36.84}$ = 10·55 tons/in².

Max. permissible bending stress when wind forces are taken into account:

F_b' = 10·0 + 25% = 12·50 tons/in².

5th to 8th Floor Beams. Mk. F1. (RS etc.)

Z for trial section = 65·59 in³

Max. Mt. due to dead and superimposed loads (D/6) = 529·3 ton-ins.

$f_b = \frac{529.3}{65.59}$ = 8·07 tons/in²

Max. Wind Mt. on 5th Fl. beam (approx., C/6) 25·48×12 = 305·76 ton-ins.

Corresponding Mt. due to dead and super. loads (D/6) = 519·0 "

Max. combined Mt. on 5th Fl. beam = 824·76 "

$f_b' = \frac{824.76}{65.59}$ = 12·56 tons/in².

1st & 2nd Floor Beams. Mk. F1 (CD & EF, Fig. 46)

Z for trial section = 90·63 in³

Max. Mt. due to dead and superimposed loads (D/6) = 540·1 ton-ins

$f_b = \frac{540.1}{90.63}$ = 5·97 tons/in²

Max. Wind Mt. on 1st Fl. beam (approx., C/5) 53·05×12 = 636·60 ton-ins.

Corresponding Mt. due to dead and super. loads (D/6) = 498·10 "

Max. combined Mt. on 1st Fl. beam = 1134·70 "

$f_b' = \frac{1134.70}{90.63}$ = 12·51 tons/in².

D/8

Check on Stresses in Columns

Internal Column Mk. A

4th Storey

Trial section consists of a 12" × 6" × 44 lb RSJ with 2/10" × ¾" plates

$A = 13{\cdot}00 + 15{\cdot}00 = 28{\cdot}00 \text{ in}^2$

$I_{xx} = 927{\cdot}06 \text{ in}^4$

$Z_{xx} = \frac{927{\cdot}06 \times 2}{13{\cdot}5} = 137{\cdot}3 \text{ in}^3$

$L = 156'' - 15''(\text{say}) = 141''$ Min $r = 0{\cdot}2(b+4) = 2{\cdot}8''$

$\frac{L}{r} = \frac{141}{2{\cdot}8} = 50{\cdot}4$

Direct load (B/9) = 173·1 tons

$f_a = \frac{173{\cdot}1}{28{\cdot}00} = 6{\cdot}18 \text{ tons/in}^2$

Max. Mt. due to dead and superimposed loads (D/6) = 213·4 ton-ins

$f_{bc} = \frac{213{\cdot}4}{137{\cdot}3} = 1{\cdot}56 \text{ tons/in}^2$

From the design chart for columns (Fig. 47) the max. permissible compressive stress due to bending:

$F_{bc} = 2{\cdot}6 \text{ tons/in}^2$

Wind Mt. (approx. from C/5) = 17·42 × 12 = 209·04 "

Max. combined Mt. on internal 4th storey column = 422·44 "

$f'_{bc} = \frac{422{\cdot}44}{137{\cdot}3} = 3{\cdot}07 \text{ tons/in}^2$

$F'_{bc} = 2{\cdot}60 + 25\% = 3{\cdot}25 \text{ tons/in}^2$

1st Storey

Trial section consists of a 12" × 8" × 65 lb. RSJ with 2/12" × 1⅛" plates

$A = 19{\cdot}12 + 27{\cdot}00 = 46{\cdot}12 \text{ in}^2$

$I_{xx} = 1653{\cdot}45 \text{ in}^4$

$Z_{xx} = \frac{1653{\cdot}45 \times 2}{14{\cdot}25} = 232{\cdot}02 \text{ in}^3$

$L = 204'' - 18''(\text{say}) = 186''$ Min. $r = 0{\cdot}2(b+4) = 3{\cdot}2''$

$\frac{L}{r} = \frac{186}{3{\cdot}2} = 58{\cdot}2$

Direct load (B/10) = 266·97 tons.

$f_a = \frac{266{\cdot}97}{46{\cdot}12} = 5{\cdot}79 \text{ tons/in}^2$

Max. Mt. due to dead and superimposed loads (D/6) = 194·2 ton-ins

$f_{bc} = \frac{194{\cdot}2}{232{\cdot}02} = 0{\cdot}84 \text{ tons/in}^2$

From design chart for columns, the max permissible compressive stress due to bending:

$F_{bc} = 2{\cdot}8 \text{ tons/in}^2$

Wind Mt. (approx from C/5) = 44·31 × 12 = 531·72 "

Max combined Mt. on internal 1st storey column = 725·92 "

$f'_{bc} = \frac{725{\cdot}92}{232{\cdot}02} = 3{\cdot}13 \text{ tons/in}^2$

$F'_{bc} = 2{\cdot}80 + 25\% = 3{\cdot}5 \text{ tons/in}^2$

D/9

Welded Frame Joints

8th Floor Beams Mk F1.

Total vertical shear at joint, (approx from B/19) = 14·73 tons

Employing 2/3½" × ½" flats in web connection; and a shear stress in ⅜" fillet welds of 6·5 tons/in²., length of fillet weld required

$$= \frac{14{\cdot}73}{0{\cdot}707 \times 0{\cdot}38 \times 6{\cdot}5} + 2/0{\cdot}38'' \text{ (say)}$$

$$= \quad 9{\cdot}25'' \text{ (min.)}$$

this is provided by 2-flats 6" in length

Mt. at joint due to dead and super. loads (D/6) = 471·7 ton-ins.

Max. Wind Mt. (approx from C/5) = 79·44 "

Max combined Mt. = 551·14 "

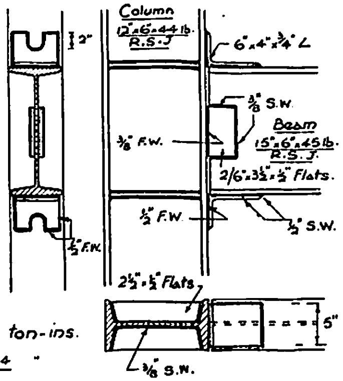

Fig. 48

Since wind moment is less than 25% of normal frame moment on beam due to dead and super. loads, the flange connections are designed for the normal frame moment of 471·7 ton-ins at a stress on fillet welds of 6·5 tons/in².

$$\therefore \text{Flange force} = \frac{471{\cdot}7}{15} = 31{\cdot}5 \text{ tons.}$$

Employing ½"-welds, length of weld requ'd

$$= \frac{31{\cdot}5}{0{\cdot}707 \times 0{\cdot}50 \times 6{\cdot}5} + 2/0{\cdot}5'' \text{ (say)} \quad = \quad 14{\cdot}7'' \text{ (min.)}$$

With 6" × 4" × ¾" ∠'s as flange cleats, length available for welding:

cleat-to-column 2/4" + 5" + 2/2" = 17" (see Fig. 48)

cleat-to-beam 2/5" + 5" = 15"

3rd Floor Beams. Mk. F1.

Web connections as above

Mt. at joint due to dead and super. loads (D/6) = 517·4 ton-ins.

Max. Wind Mt (approx from C/5) = 436·8 "

Max combined Mt. = 954·2 "

$$\text{Flange force} \ \frac{954{\cdot}2}{16} = 59{\cdot}3 \text{ tons}$$

Length of ½" Fillet weld

$$\text{requ'd} = \frac{59{\cdot}3}{0{\cdot}707 \times 0{\cdot}50 \times (6{\cdot}5 + 25\%)} + 2/0{\cdot}5'' \text{ (say)}$$

$$= \quad 21{\cdot}7'' \text{ (min)}$$

With split-beam flange-cleats out of a 16" × 6" × 62 lb RSJ length available for welding.

cleat-to-column 2/6" + 9" + 2/3" = 27"

cleat-to-beam 2/6·5" + 6" + 2/4" = 27"

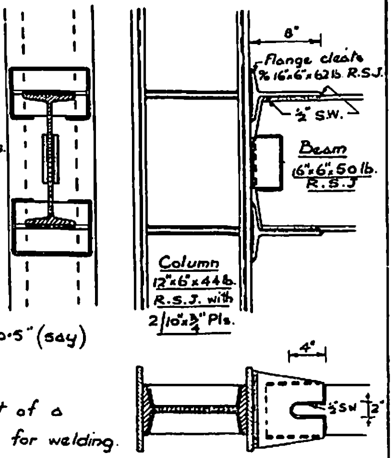

Fig. 49.

this weight is 15·7 tons indicating an overall saving of 10% in the steelwork. The rigid method of design and the welding of the frame joints each account for approximately half of this saving in steel for the building frame considered.

Further Study.—Several points arising out of the rigid frame analysis are suggested for further study :—

(*a*) The frame considered is symmetrical and subjected to symmetrical loading so that the problem of side sway has not arisen.

(*b*) It is impracticable in design to consider all the variations that might occur in the arrangement of floor loads in order to find the critical conditions for each storey length of the columns, although certain alternative arrangements might be investigated.

(*c*) In Design Example (D), although the superimposed loading on the columns is reduced from floor to floor below the 7th in accordance with established practice (see B.S. 449, C1. 8) the frame moments in all beams and columns are computed from the end moments due to the full design loads on each floor (see Figs. 45 and 46).

(*d*) The rigid frame analysed is highly indeterminate so that if any member of it is over-stressed there is no immediate danger of the frame collapsing. Overstress in any beam would result in the formation of "plastic" hinges at the critical sections and cause a stress redistribution throughout the framework. The beam, meanwhile, would not fail at once but would change its mode of supporting its loads. Such considerations as these have been advanced in advocating increased permissible design stresses for rigid frames(2).

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55. ,, ,, ,, (Square Wire).

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66. ,, ,, ,, ,, (Tangential Stress) } Connected.
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